Symmetries in Quantum Mechanics

Final Exam — Monday January 16, 2012

1. Symmetries is Quantum Mechanics: what are they good for?

2. Say we have an orthonormal set of three states $\{\psi_x, \psi_y, \psi_z\}$ (say of some atom), on which rotations act in the standard vector representation, i.e. they transform among each other in the same way as a vector (x, y, z) in $\mathbb{R}^3$.

   (a) How do the operators corresponding to angular momentum L_x, L_y and L_z act on these states?

   (b) What are the possible values of L_z?

   (c) If we prepare a beam of these atoms, prepared to be in states restricted to be linear combinations of the above three states, and we first pass this beam through a filter that allows through only atoms with maximally positive spin in the z-direction, next through a filter that allows only atoms with maximally positive spin in the *x-direction*, and finally a filter that allows only atoms with maximally *negative* spin in the z-direction, then what fraction of the original beam will survive?

   (d) Can this result change if the beam passes through some homogeneous magnetic field between two subsequent detectors?

3. (a) A spin 1 particle A at rest decays into a spin 1/2 particle B and a spin 1/2 particle C. What are the possible final values of the z-components of spin of B and C assuming the final (center of mass) orbital angular momentum is measured to be zero?

   (b) What are the other possible values of the final orbital angular momentum that could be measured, and what are the corresponding values of the spins of B and C?

   (c) How do the possibilities get further reduced if you know the intrinsic parities of A, B and C are all even?

   (d) Restricting again to the zero orbital angular momentum sector, and assuming all initial spin states are equally likely, what are the probabilities for the z-components of the spin of B and C?

   (e) How would you solve this last problem for a spin 3 particle A decaying into a spin 1 particle B and a spin 2 particle C? (Optionally: solve it.)