Oplossingen Oefenzitting 5

1

4. A toroid with a mean radius of 20.0 cm and 630 turns is filled with powdered steel whose magnetic susceptibility χ is 100. The current in the windings is 3.00 A. Find B (assumed uniform) inside the toroid.

Solution: Assuming a uniform B inside the toroid is equivalent to assuming $r \ll R$ (see Fig. 4); then $B_0 = \mu_0 H \approx \mu_0 N I$ as for a tightly wound solenoid. This leads to $B_0 = \mu_0 \frac{630 \cdot 3.00}{2\pi \cdot 0.200} = 0.00189$ T. With the steel, $B = (1+\chi)\mu_0 H = 101 \cdot 0.00189$ T $= 0.191$ T.

2

3. A solenoid 2.50 cm in diameter and 30.0 cm long has 300 turns and carries 12.0 A. *(i)* Calculate the flux through the surface of a disk of radius 5.00 cm that is positioned perpendicular to and centered on the axis of the solenoid, as shown in Fig. 3 (a). *(ii)* Figure 3 (b) shows an enlarged end view of the same solenoid. Calculate the flux through the blue area, which is defined by an annulus that has an inner radius of 0.400 cm and outer radius of 0.800 cm.

Solution $\Phi_B = \vec{B} \cdot \vec{A} = BA$, where A is the cross-sectional area of the solenoid, $\Phi_B = \frac{\mu_0 N I}{L} \pi r^2 =$ 7.40 μWb. *(ii)* $\Phi_B = \vec{B} \cdot \vec{A} = BA = \frac{\mu_0 N I}{L} \pi \left(r_2^2 - r_1^2\right) = 2.27$ μWb.

3

1. Consider the hemispherical closed surface in Fig. 1. The hemisphere is in a uniform magnetic field that makes an angle θ with the vertical. Calculate the magnetic flux through *(i)* the flat surface S_1 and *(ii)* the hemispherical surface S_2.

Solution *(i)* $\Phi_B|_{\text{flat}} = \vec{B} \cdot \vec{A} = B\pi R^2 \cos(\pi - \theta) = -B\pi R^2 \cos\theta$. *(ii)* The net flux out of the closed surface is zero: $\Phi_B|_{\text{flat}} + \Phi_B|_{\text{curved}} = 0$, hence $\Phi_B|_{\text{curved}} = B\pi R^2 \cos\theta$.

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Oplossingen Oefenzitting 6

1

This is a tough problem because there are two current loops, and each sliding rail acts like a battery (a source of emf). So there's no way to avoid using Kirchhoff's laws:

Name the currents as shown in the diagram:

Left loop: $+Bdv_2 - I_2R_2 - I_1R_1 = 0$

Right loop: $+Bdv_3 - I_3R_3 + I_1R_1 = 0$

At the junction: $I_2 = I_1 + I_3$

Then, $Bdv_2 - I_1R_2 - I_3R_2 - I_1R_1 = 0$

$$I_3 = \frac{Bdv_3}{R_3} + \frac{I_1R_1}{R_3}.$$

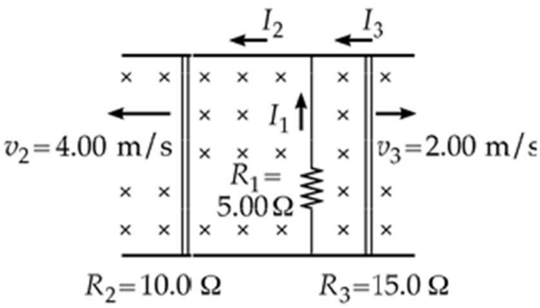


FIG. P31.31

So, $$Bdv_2 - I_1(R_1 + R_2) - \frac{Bdv_3R_2}{R_3} - \frac{I_1R_1R_2}{R_3} = 0$$

$$I_1 = Bd\left(\frac{v_2R_3 - v_3R_2}{R_1R_2 + R_1R_3 + R_2R_3}\right) \text{ upward}$$

$$I_1 = (0.0100\text{ T})(0.100\text{ m})\left[\frac{(4.00\text{ m/s})(15.0\,\Omega) - (2.00\text{ m/s})(10.0\,\Omega)}{(5.00\,\Omega)(10.0\,\Omega) + (5.00\,\Omega)(15.0\,\Omega) + (10.0\,\Omega)(15.0\,\Omega)}\right] = \boxed{145\ \mu\text{A}} \text{ upward.}$$

2

5. Two parallel rails with negligible resistance are 10.0 cm apart and are connected by a 5.00-Ω resistor. The circuit also contains two metal rods having resistances of 10.0 Ω and 15.0 Ω sliding along the rails (Fig. 5). The rods are pulled away from the resistor at constant speeds of 4.00 m/s and 2.00 m/s, respectively. A uniform magnetic field of magnitude 0.01 T is applied perpendicular to the plane of the rails. Determine the current in the 5.00-Ω resistor.

Solution Name the currents as shown in Fig. 5: left loop, $+Bdv_2 - I_2R_2 - I_1R_1 = 0$; right loop, $+Bdv_3 - I_3R_3 + I_1R_1 = 0$; and at the junction, $I_2 = I_1 + I_3$ Then, $Bdv_2 - I_1R_2 - I_3R_2 - I_1R_1 = 0$, with $I_3 = \frac{Bdv_3}{R_3} + \frac{I_1R_1}{R_3}$. Hence $Bdv_2 - I_1(R_1 + R_2) - \frac{Bdv_3R_2}{R_3} - \frac{I_1R_1R_2}{R_3} = 0$, yielding $I_1 = Bd\,\frac{v_2R_3 - v_3R_2}{R_1R_2 + R_1R_3 + R_2R_3} = 145\ \mu\text{A}$ upward.

3

1. A semicircular conductor of radius $R = 0.250$ m is rotated about the axis AC at a constant rate of 120 rev/min (Fig. 1). A uniform magnetic field in all of the lower half of the figure is directed out of the plane of rotation and has a magnitude of 1.30 T. *(i)* Calculate the maximum value of the emf induced in the conductor. *(ii)* What is the value of the average induced emf for each complete rotation? *(iii)* How would the answers to (i) and (ii) change if B were allowed to extend a distance R above the axis of rotation? Sketch the emf versus time *(iv)* when the field is as drawn in Fig. 1 and *(v)* when the field is extended as described in (iii).

Solution *(i)* $\varepsilon_{\max} = BA\omega = B\pi R^2\omega/2 = 1.30\ \text{T} \cdot \frac{\pi}{2} \cdot (0.25\ \text{m})^2 \cdot 4.00\pi\ \text{rad/s} = 1.60\ \text{V}$. *(ii)* $\bar{\varepsilon} = \frac{1}{2\pi}\int_0^{2\pi} \varepsilon\ d\theta = \frac{BA\omega}{2\pi}\int_0^{2\pi} \sin\theta\ d\theta = 0$. *(iii)* The maximum and average ε would remain unchanged. *(iv)* See Fig. 2. *(v)* See Fig. 2.

6.A uniform but time-varying magnetic field B(t) exists in a circular region of radius a and is directed into the plane of the paper, as showm in the figure. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region. [2000-2 marks]

Ans.

(b) Magnetic field B(t) is directed into plane of the paper.

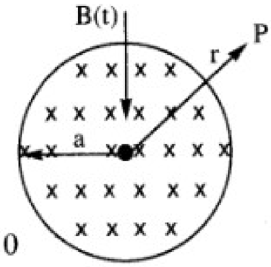
$$\oint \vec{E}.\vec{dl} = \frac{d\phi}{dt}$$

P lies outside the field.

or $E(2\pi r) = \frac{d}{dt}(\vec{B}.\vec{A})$

or $2\pi r E = \pi a^2 \left(\frac{dB}{dt}\right) \cos 0$

or $E = \frac{a^2}{2r}\frac{dB}{dt}$ *or* $E \propto \frac{1}{r}$



+

F = E * q

➔ einduitkomst F = 8 * 10^{-21}N

8. Two infinitely long solenoids (seen in cross section) pass through a circuit as shown in Fig. 8. The magnitude of B inside each is the same and is increasing at the rate of 100 T/s. What is the current in each resistor?

Solution In the loop on the left, the induced emf is $|\varepsilon| = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi(0.10 \text{ m})^2\, 100 \text{ T/s} = \pi \text{ V}$ and it attempts to produce a counterclockwise current in this loop. In the loop on the right, the

induced emf is $|\varepsilon| = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi(0.15 \text{ m})^2\, 100 \text{ T/s} = 2.25\ \pi \text{ V}$ and it attempts to produce a clockwise current. Assume that I_1 flows down through the 6.00 Ω resistor, I_2 flows down through the 5.00 Ω resistor, and that I_3 flows up through the 3.00 Ω resistor. From Kirchhoffs junction rule: $I_3 = I_1 + I_2$. Using the loop rule on the left loop: $6.00I_1 + 3.00I_3 = \pi$. Using the loop rule on the right loop: $5.00I_2 + 3.00I_3 = 2.25\pi$ Solving these three equations simultaneously, $I_1 = 0.062$ A, $I_2 = 0.860$ A, and $I_3 = 0.923$ A.

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2. In the circuit shown in Fig. 1, let $L = 7.00$ H, $R = 9.00\ \Omega$, and $\varepsilon = 120$ V. What is the self-induced emf 0.200 s after the switch is closed?

Solution Duplicating the procedure of problem 1 we obtain $I = \frac{\varepsilon}{R}(1 - e^{-t/\tau})$. Since $\tau = L/R = 0.78$ s, we have $I = \frac{120\ \mathrm{V}}{9.00\ \Omega}(1 - e^{-0.26}) = 3.05$ A. Now $\Delta V_R = IR = 27.4$ V and $\Delta V_L = \varepsilon - \Delta V_R = 92.6$ V.

3

21. We create an Amperean loop of radius r to calculate the magnetic field within the wire using Eq. 28-3. Since the resulting magnetic field only depends on radius, we use Eq. 30-7 for the energy density in the differential volume $dV = 2\pi r\ell dr$ and integrate from zero to the radius of the wire.

$$\oint \vec{\mathbf{B}}\cdot d\vec{\ell} = \mu_0 I_{enc} \to B(2\pi r) = \mu_0\left(\frac{I}{\pi R^2}\right)(\pi r^2) \to B = \frac{\mu_0 I r}{2\pi R^2}$$

$$\frac{U}{\ell} = \frac{1}{\ell}\int u_B dV = \int_0^R \frac{1}{2\mu_0}\left(\frac{\mu_0 I r}{2\pi R^2}\right)^2 2\pi r dr = \frac{\mu_0 I^2}{4\pi R^4}\int_0^R r^3 dr = \boxed{\frac{\mu_0 I^2}{16\pi}}$$

4

26. (*a*) At the moment the switch is closed, no current will flow through the inductor. Therefore, the resistors R_1 and R_2 can be treated as in series.

$$\mathscr{E} = I(R_1 + R_2) \to \boxed{I_1 = I_2 = \frac{\mathscr{E}}{R_1 + R_2},\ I_3 = 0}$$

(*b*) A long time after the switch is closed, there is no voltage drop across the inductor so resistors R_2 and R_3 can be treated as parallel resistors in series with R_1.

$$I_1 = I_2 + I_3, \quad \mathscr{E} = I_1 R_1 + I_2 R_2, \quad I_2 R_2 = I_3 R_3$$

$$\frac{\mathscr{E} - I_2 R_2}{R_1} = I_2 + \frac{I_2 R_2}{R_3} \to \boxed{I_2 = \frac{\mathscr{E} R_3}{R_2 R_3 + R_1 R_3 + R_1 R_2}}$$

$$I_3 = \frac{I_2 R_2}{R_3} = \boxed{\frac{\mathscr{E} R_2}{R_2 R_3 + R_1 R_3 + R_1 R_2}} \qquad I_1 = I_2 + I_3 = \boxed{\frac{\mathscr{E}(R_3 + R_2)}{R_2 R_3 + R_1 R_3 + R_1 R_2}}$$

(*c*) Just after the switch is opened the current through the inductor continues with the same magnitude and direction. With the open switch, no current can flow through the branch with the switch. Therefore the current through R_2 must be equal to the current through R_3, but in the opposite direction.

$$I_3 = \boxed{\frac{\mathscr{E} R_2}{R_2 R_3 + R_1 R_3 + R_1 R_2}}, \quad I_2 = \boxed{\frac{-\mathscr{E} R_2}{R_2 R_3 + R_1 R_3 + R_1 R_2}}, \quad I_1 = \boxed{0}$$

(*d*) After a long time, with no voltage source, the energy in the inductor will dissipate and no current will flow through any of the branches.

$$I_1 = I_2 = I_3 = \boxed{0}$$

OF:

3. The switch in Fig. 2 is open for $t < 0$ and then closed at time $t = 0$. Find the current in the inductor and the current in the switch as functions of time thereafter.

Solution Name the currents as shown in Fig. 2. Using Kirchhoff's laws we obtain $I_1 = I_2 + I_3$, $10.0\text{ V} - 4.00 I_1 - 4.00 I_2 = 0$, and $10.0\text{ V} - 4.00 I_1 - 8.00 I_3 - 1.00\frac{dI_3}{dt} = 0$. From the first two equations it follows that $10.0\text{ V} + 4.00 I_3 - 8.00 I_1 = 0$ and $I_1 = 0.50 I_+ 3 + 1.25\text{ A}$. Then the last equation can be rewritten as $10.0\text{ V} - 4.00(0.500 I_3 + 1.25\text{ A}) - 8.00 I_3 - 1.00\text{ H}\frac{dI_3}{dt} = 0$, yielding $1\text{ H}\frac{dI_3}{dt} + 10.0\ \Omega I_3 = 5.00\text{ V}$. We solve the differential equation to obtain $I_3(t) = \frac{5.00\text{V}}{10.0\ \Omega}\left[1 - e^{-10.0\ \Omega t/1.00\text{ H}}\right] = 0.50\text{ A}\left[1 - e^{-10t/s}\right]$. Then $I_1 = 1.25 + 0.50 I_3 = 1.50\text{ A} - 0.25\text{ A}e^{-10t/s}$.

5

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(Niet exact hetzelfde)

6. Two inductors having self-inductances L_1 and L_2 are connected in parallel as shown in Fig. 3(a). The mutual inductance between the two inductors is M. Determine the equivalent self-inductance L_{eq} for the system shown in Fig. 3(b).

Solution With $I = I_1 + I_2$, the voltage across the pair is: $\Delta V = -L_1\frac{dI_1}{dt} - M\frac{dI_2}{dt} = -L_2\frac{dI_2}{dt} - M\frac{dI_1}{dt} = -L_{\text{eq}}\frac{dI}{dt}$. Hence, $-\frac{dI_1}{dt} = \frac{\Delta V}{L_1} + \frac{M}{L_1}\frac{dI_2}{dt}$ and $-L_2\frac{dI_2}{dt} + \frac{M\,\Delta V}{L_1} + \frac{M^2}{L_1}\frac{dI_2}{dt} = \Delta V$, yiedling

$$(-L_1L_2 + M^2)\frac{dI_2}{dt} = \Delta V(L_1 - M). \tag{1}$$

By substitution, $-\frac{dI_2}{dt} = \frac{\Delta V}{L_2} + \frac{M}{L_2}\frac{dI_1}{dt}$ leads to

$$(-L_1L_2 + M^2)\frac{dI_1}{dt} = \Delta V(L_2 - M). \tag{2}$$

Adding (1) to (2), $(-L_1L_2 + M^2)\frac{dI}{dt} = \Delta V(L_1 + L_2 - 2M)$, and therefore $L_{\text{eq}} = -\frac{\Delta V}{dI/dt} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$.

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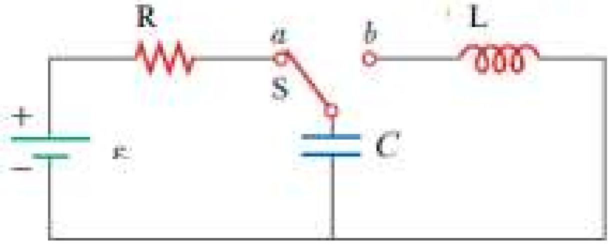
14. The switch in the figure below is connected to position *a* for a long time interval. At $t = 0$, the switch is thrown to position *b*. After this time, what are the following?



(a) the frequency of oscillation of the *LC* circuit. (b) the maximum charge that appears on the capacitor. (c) the maximum current in the inductor. (d) the total energy the circuit possesses at *t* .

(a) $\omega = \frac{1}{\sqrt{LC}}$

$\therefore \omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} \Rightarrow f = \underline{\frac{1}{2\pi\sqrt{LC}}}$

(b) Maximum charge = charge in C just before the switch is thrown to position b
$= C\varepsilon$

(c) Consivation of energy :

$$\frac{1}{2}\frac{Q_{max}^2}{C} = \frac{1}{2}LI_{max}^2 \Rightarrow I_{max}^2 = \frac{Q_{max}^2}{LC}$$

$$\Rightarrow I_{max} = \underline{\frac{Q_{max}}{\sqrt{LC}}} \text{ or } \frac{C\varepsilon}{\sqrt{LC}} = \underline{\varepsilon\sqrt{\frac{C}{L}}}$$

(d) Consivation of energy ⇒ Total energy is constant, independent of time.

$$\therefore U = \frac{1}{2}\frac{Q_{max}^2}{C} = \frac{1}{2}\frac{(C\varepsilon)^2}{C} = \underline{\frac{1}{2}C\varepsilon^2}$$

3

33. (*a*) We write the oscillation frequency in terms of the capacitance using Eq. 30-14, with the parallel plate capacitance given by Eq. 24-2. We then solve the resulting equation for the plate separation distance.

$$2\pi f = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{L(\varepsilon_0 A/x)}} \rightarrow x = \boxed{4\pi^2 A\varepsilon_0 f^2 L}$$

(*b*) For small variations we can differentiate *x* and divide the result by *x* to determine the fractional change.

$$dx = 4\pi^2 A\varepsilon_0 (2 f df) L \;;\; \frac{dx}{x} = \frac{4\pi^2 A\varepsilon_0 (2 f df) L}{4\pi^2 A\varepsilon_0 f^2 L} = \frac{2df}{f} \rightarrow \boxed{\frac{\Delta x}{x} \approx \frac{2\Delta f}{f}}$$

(*c*) Inserting the given data, we can calculate the fractional variation on *x*.

$$\frac{\Delta x}{x} \approx \frac{2(1\ \text{Hz})}{1\ \text{MHz}} = 2\times 10^{-6} = \boxed{0.0002\%}$$

4

10. The energy of an RLC circuit decreases by 1.00% during each oscillation when $R = 2.00\ \Omega$. If this resistance is removed, the resulting LC circuit oscillates at a frequency of 1.00 kHz. Find the values of the inductance and the capacitance.

Solution The period of damped oscillation is $T = \frac{2\pi}{\omega_d}$. After one oscillation the charge returning to the capacitor is $Q = Q_{\max} e^{-\frac{RT}{2L}} = Q_{\max} e^{-\frac{2\pi R}{2L\omega_d}}$. The energy is proportional to the charge squared,so after one oscillation it is $\mathcal{U} = \mathcal{U}_0 e^{-\frac{2\pi}{L\omega_d}}$. Then $e^{\frac{2\pi R}{L\omega_d}} = \frac{1}{0.99}$, which leads to $\frac{2\pi 2\Omega}{L\omega_d} = \ln(1.0101) = 0.001005$. It follows that $L\omega_d = \frac{2\pi 2\Omega}{0.001005} = 1250\Omega = L\left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)^{1/2}$, yielding $1.563\times 10^6\Omega^2 = \frac{L}{C} - \frac{(2\Omega)^2}{4}$ or equivalently $\frac{L}{C} = 1.563\times 10^6\Omega^2$. We are also given $\omega = 2\pi\times 10^3/\text{s} = \frac{1}{\sqrt{LC}}$, which leads to $LC = \frac{1}{(2\pi\times 10^3/\text{s})^2} = 2.533\times 10^{-8}\ \text{s}^2$. Solving simultaneously, $C = 2.533\times 10^{-8}\ \text{s}^2/L$, yields $\frac{L^2}{2.533\times 10^{-8}\ \text{s}^2} = 1.563\times 10^6\ \Omega^2$; therefore $L = 0.199$ H and $C = \frac{2.533\times 10^{-8}\ \text{s}^2}{0.199\ \text{H}} = 127$ nF.

5

Oplossingen Oefenzitting 9

1

(gelijkaardige oefening, niet iedentiek)

3. *(i)* For the series RLC connection of Fig. 4, draw a phasor diagram for the voltages. The amplitudes of the voltage drop across all the circuit elements involved should be represented with phasors. *(ii)* An RLC circuit consists of a 150-Ω resistor, a 21-μF capacitor and a 460-mH inductor, connected in series with a 120-V, 60-Hz power supply. What is the phase angle between the current and the applied voltage? *(iii)* Which reaches its maximum earlier, the current or the voltage?

Solution *(i)* For the series connection, the instantaneous voltage across the system is equal to the sum of voltage across each element. The phase angle between the voltage (across the system) and the current (through the system) is: $\phi = \arctan\frac{X_L - X_C}{R}$. In Fig. 5 the phasor diagram for a series RLC circuit is shown for both the inductive case $X_L > X_C$ and the capacitive case $X_L < X_C$. On the one hand, in the inductive case, $V_{0,L} > V_{0,c}$, we see that $\vec{V}_0$ leads $\vec{I}_0$ by a phase ϕ. On the other hand, in the capacitance case, $V_{0,C} > V_{0,L}$, we have that $\vec{I}_0$ leads $\vec{V}_0$ by a phase ϕ. *(ii)* From the definition, the inductive reactance of the inductor is $X_L = \omega L = 2\pi \cdot 60\ \text{Hz} \cdot 0.46\ \text{H} = 173\ \Omega$. From the definition, the capacitive reactance of the capacitor is $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \cdot 60\ \text{Hz} \cdot 21\times10^{-6}\ \text{F}} = 126\ \Omega$. The phase angle between the voltage (across the system) and the current (through the system) is: $\phi = \arctan\frac{X_L - X_C}{R} = \arctan\frac{174\ \Omega - 126\ \Omega}{150\ \Omega} = 17.4°$ *(iii)* The voltage leads the current.

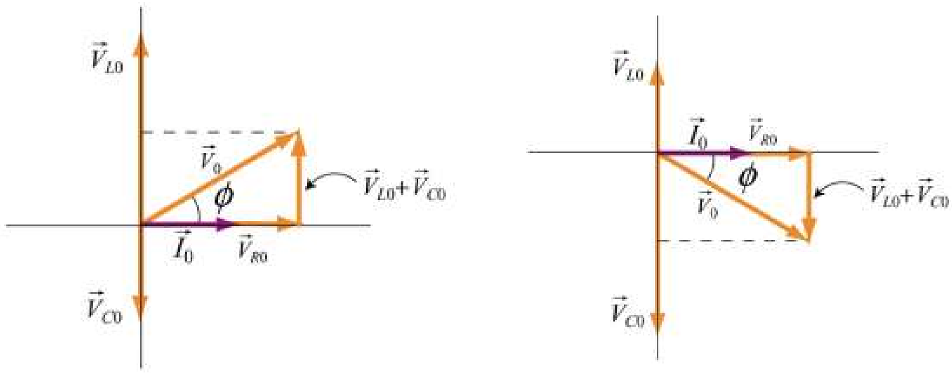


Figure 5: Phasor diagram for the series RLC circuit for $X_L > X_C$ (left) and $X_L < X_C$ (right).

2

55. (*a*) The rms current is the rms voltage divided by the impedance. The impedance is given by Eq. 30-28a with no capacitive reactance.

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi fL)^2} \ .$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + 4\pi^2 f^2 L^2}} = \frac{120\,\text{V}}{\sqrt{(965\,\Omega)^2 + 4\pi^2 (60.0\,\text{Hz})^2 (0.225\,\text{H})^2}}$$

$$= \frac{120\,\text{V}}{968.7\,\Omega} = \boxed{0.124\,\text{A}}$$

(*b*) The phase angle is given by Eq. 30-29a with no capacitive reactance.

$$\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{2\pi fL}{R} = \tan^{-1}\frac{2\pi(60.0\,\text{Hz})(0.225\,\text{H})}{965\,\Omega} = \boxed{5.02^\circ}$$

The current is lagging the source voltage.

(*c*) The power dissipated is given by $P = I_{\text{rms}}^2 R = (0.124\,\text{A})^2 (965\,\Omega) = \boxed{14.8\,\text{W}}$

(*d*) The rms voltage reading is the rms current times the resistance or reactance of the element.

$$V_{\text{rms}_R} = I_{\text{rms}} R = (0.124\,\text{A})(965\,\Omega) = 119.7\,\text{V} \approx \boxed{120\,\text{V}}$$

$$V_{\text{rms}_L} = I_{\text{rms}} X_L = I_{\text{rms}} 2\pi fL = (0.124\,\text{A}) 2\pi (60.0\,\text{Hz})(0.25\,\text{H}) = \boxed{10.5\,\text{V}}$$

Note that, because the maximum voltages occur at different times, the two readings do not add to the applied voltage of 120 V.

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