Symmetries in Quantum Mechanics

Final Exam — Friday January 20, 2012

1. The Wigner-Eckart theorem: what's it good for? (in a few lines)

2. Under what circumstances is it possible to simultaneously measure with arbitrary precision energy and (*i*) position, (*ii*) momentum, (*iii*) angular momentum and (*iv*) velocity, of a particle in a static electric field? And in a static magnetic field?

3. A particle is in a state with wave function $\psi(x, y, z) = (x + y)f(r)$, where $r = \sqrt{x^2 + y^2 + z^2}$.

   (a) Argue using simple symmetry reasoning that $\langle\chi|\psi\rangle = 0$ for any $|\chi\rangle$ that is even under parity or odd under the exchange of x and y coordinates.

   (b) What are the possible outcomes for measurements of L^2 and L_z, and with what probabilities?

   (c) Let $|n\ell m\rangle$ be a hydrogen wave function. For which values of ℓ, m is it guaranteed that $\langle n\ell m|z|\psi\rangle = 0$, for which that $\langle n\ell m|x^2 - y^2|\psi\rangle = 0$, and for which that $\langle n\ell m|L_+|\psi\rangle = 0$? (Here $L_+ = L_x + iL_y$.)

4. A system consists of 100 spin 1 particles. Can you explicitly construct a state with total spin quantum numbers $(J, M) = (100, 99)$?

5. (a) Two atoms are slowly brought towards each other. What will happen to their energy spectra?

   (b) Three Helium-3 atoms walk into a bar. All energy levels of this trio are doubly degenerate. What can be done to undo their doubling?