Exam Advanced Quantum Mechanics
16 January 2017

Name:.....

- Please write your answers on numbered pages. Write your name on each page. Start a separate page for each new question. Additional pages with your draft work, rough calculations or incomplete answers are handed in separately but are not considered.
- The exam is oral, closed book

1. Suppose we have a two-level system with time-dependent Hamiltonian $H = H_0 + V(t)$ of the form

$$H_0 = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}, \quad V(t) = \begin{pmatrix} 0 & \delta\, e^{i\omega t} \\ \delta\, e^{-i\omega t} & 0 \end{pmatrix}$$

with $\omega_{21} = (E_2 - E_1)/\hbar \neq 0$. Write the eigenfunctions of H_0 as $|1\rangle$ and $|2\rangle$, $H_0|1\rangle = E_1|1\rangle$ and $H_0|2\rangle = E_2|2\rangle$. The wave function at time $t \geq 0$ for the full system is written as

$$|\psi_t\rangle = c_1(t)|1\rangle + c_2(t)|2\rangle$$

Take the initial condition $c_1(0) = 1, c_2(0) = 0$. Show that

$$|c_2(t)|^2 = \frac{\delta^2}{\delta^2 + \hbar^2(\omega - \omega_{21})^2/4}\, \sin^2 \Omega t$$

for so called Rabi frequency $\Omega := \left(\frac{\delta^2}{\hbar^2} + (\omega - \omega_{21})^2/4\right)^{1/2}$.
How does the Jaynes-Cummings model improve on that model?

2. Consider the coherent states from quantum optics,

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}\, |n\rangle$$

for $\alpha \in \mathbb{C}$. Show that a coherent state is a displaced vacuum state in the sense that for displacement operator

$$D(\alpha) = \exp(\alpha a^* - \alpha^* a)$$

we have

$$|\alpha\rangle = D(\alpha)\,|0\rangle$$

Prove also that

$$D(\alpha)\,D(\beta) = D(\alpha + \beta)$$

when $\alpha, \beta \in \mathbb{R}$.
Use the Baker-Campbell-Hausdorff formula in the form

$$D(\alpha) = e^{-|\alpha|^2/2}\, e^{\alpha a^*}\, e^{-\alpha^* a}$$

3. Give the experimental set-up and facts concerning the Aharanov-Bohm effect applied to the two-split experiment?

4. Obtain in the first Born approximation the scattering amplitude, the differential and the total cross-sections for scattering by the Yukawa potential $V(r) = V_0 \exp(-\alpha r)/r$.

5. Show that the Schrödinger equation is time-reversal invariant when the potential V is real.

6. Recent and not so recent experiments have shown that the Bell inequalities are violated. What does that mean?