Exam Advanced Quantum Mechanics
19 August 2016 AM

Name:..................

- Please write your answers on numbered pages. Write your name on each page. Start a separate page for each new question. Additional pages with your draft work, rough calculations or incomplete answers are handed in separately but are not considered.
- The exam is oral, closed book

1. [oral] The most general density matrix for a spin 1/2 system is

$$\rho = \frac{1}{2}(1 + a \cdot \sigma)$$

where a is a vector whose length is not greater than 1, and σ is the vector of the three Pauli matrices.
If the system has a magnetic moment $\mu = \gamma\hbar\sigma/2$ and is in a constant magnetic field B, calculate the time-dependent density matrix $\rho(t)$ in terms of the polarization vector a_t in

$$\rho(t) = \frac{1}{2}(1 + a_t \cdot \sigma)\,.$$

2. [oral] Consider the coherent states from quantum optics,

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}\,|n\rangle$$

for $\alpha \in \mathbb{C}$. Show that a coherent state is a displaced vacuum state in the sense that for displacement operator

$$D(\alpha) = \exp(\alpha a^* - \alpha^* a)$$

we have

$$|\alpha\rangle = D(\alpha)\,|0\rangle$$

Use the Baker-Campbell-Hausdorff formula to simplify

$$D(\alpha) = e^{-|\alpha|^2/2}\, e^{\alpha a^*}\, e^{-\alpha^* a}$$

Check that the probability to have k photons is given by

$$\text{Prob}[N = k] = e^{-|\alpha|^2}\,\frac{|\alpha|^{2k}}{k!}$$

3. [oral] What is the issue or the relevance of the Bell inequalities?

4. [oral] What is the Aharanov-Bohm effect?

5. Obtain in the first Born approximation the scattering amplitude, the differential and the total cross-sections for scattering by the Yukawa potential $V(r) = V_0\,\exp(-\alpha r)/r$.

6. Apply the gauge transformation generated by taking

$$\chi(\vec{r}, t) = -\frac{1}{2} B\, x\, y$$

to the potentials $\vec{A}(\vec{r}) = \frac{1}{2}(\vec{B} \times \vec{r})$, $\phi = 0$, where $\vec{B}$ is taken along the z-axis. Show that the transformed time-independent Schrödinger equation, for a spinless particle of charge $q = -e$ and mass m, is

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + \frac{e\, i\hbar B y}{2m}\,\frac{\partial}{\partial x} + \frac{e^2 B^2 y^2}{2m} \right)\,\Phi(\vec{r}) = E\,\Phi(\vec{r})$$