Exam Advanced Quantum Mechanics
25 August 2017

Name:.....

- Please write your answers on numbered pages. Write your name on each page. Start a separate page for each new question. Additional pages with your draft work, rough calculations or incomplete answers are handed in separately but are not considered.
- The exam is oral, closed book

Oral: Give clear and short answers to the following questions. Use drawings and formulae to explain better your words.

1. a) What was the purpose of the Einstein-Podolsky-Rosen paper (1935) and what is the (possibly simplified) argument?
b) Explain the difference between superposition and mixture.
c) What is an entangled state or system? Give a specific example to illustrate your point.
d) Recent and not so recent experiments have shown that the Bell inequalities are violated. What does that mean?

2. Show that the Schrödinger equation is time-reversal invariant when the potential V is real.

3. Explain how the first Born approximation for the cross section can be obtained from the Lippmann-Schwinger equation:

$$\psi_{\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} - \frac{1}{4\pi}\int \frac{\exp[ik|\vec{r}-\vec{u}|]}{|\vec{r}-\vec{u}|}\, U(\vec{u})\, \psi_{\vec{k}}(\vec{u})\, \mathrm{d}\vec{u}$$

Explain all the notation in that equation.

Written: write clearly.

3. Calculate for a complex number z,

$$e^{za^*}\, e^{-z^*a}\, |0\rangle$$

where, respectively, a and a^* are the annihilation and creation operator for the harmonic oscillator with ground state $|0\rangle$. Call $|n\rangle$ the eigenstate with n particles/photons.

4. The most general density matrix for a spin 1/2 system is

$$\rho = \frac{1}{2}(1 + a \cdot \sigma)$$

where a is a vector whose length is not greater than 1, and σ is the vector of the three Pauli matrices.
If the system has a magnetic moment $\mu = \gamma\hbar\,\sigma/2$ and is in a constant magnetic field B, calculate the time-dependent density matrix $\rho(t)$ in terms of the polarization vector a_t in

$$\rho(t) = \frac{1}{2}(1 + a_t \cdot \sigma)\,.$$

5. Determine the Clebsch-Gordan coefficients associated with the addition of one spin 1/2 and one spin 1.