analytical mechanics
1 September 2016 AM
Name: ..... .

1. Write down the Lagrangian for the following system: a cart of mass m can roll without friction on a rail along the x-axis. A pendulum, consisting of a stick of length ℓ and a point mass m, is mounted rigidly on the cart and can move freely within the $x - z$ vertical plane.

2. Give the Liouville equation for the smooth dynamical system $\dot{x}(t) = f(x(t)), x(t) \in \mathbb{R}^n$.

3. Present a derivation of the Hamilton-Jacobi equation.

4. Show that the logistic map
$x \mapsto r\,x(1-x)$ on $[0,1]$
has a two-cycle for all $r > 3$ and discuss its stability.
Show that the logistic map for $r = 4$ is conjgate to the tent map.

5. Show that the periodic points of the Bernoulli shift
$x \mapsto 2x \bmod 1$ on $[0,1]$
are dense in $[0,1]$.

6. Consider the motion of a particle in one dimension under a potential $V(x) = -kx^2/2 + kx^4/(4a^2)$ function of parameter $a > 0$.
a) Draw the possible orbits in phase space (x, p) [phase portrait].
b) Show that the derivative with respect to the energy of the integral $\oint p dx$ over one period, equals the period of the motion.