Examen
Statistische Mechanica bij Evenwicht
19 December 2013, 10u30

The score is calculated to 20 points!

2.5 points

Quantum statistics

Consider a quantum system with two non-interacting particles which can be each in 4 different quantum states. A ground state with energy 0 and 3 degenerate states with energy ε.

Calculate the canonical partition functions $Z(T)$ in the case that the two particles are two identical Bosons or two identical Fermions.

2.5 points

Low temperature limit for pressure

Show that in a system of non-interacting Bosons the pressure vanishes in the limit of vanishing temperature $T \to 0$.

2.5 points

Fermions wavefunctions

We consider three identical non-interacting fermions in three different energy levels described by the wavefunctions $\phi_k(\vec{q})$, $\phi_m(\vec{q})$ and $\phi_l(\vec{q})$. From these three determine the three particles wavefunction $\Psi(\vec{q}_1, \vec{q}_2, \vec{q}_3)$.

2.5 points

Ground state Fermi gas

Consider a Fermi gas of free particles at temperature $T = 0$ K enclosed in a cubic box of volume V. The density of states is

$$g(\varepsilon) = \frac{V}{4\pi^2} \frac{(2m)^{3/2}}{\hbar^3} \sqrt{\varepsilon}$$

Find the ground state energy E_0 of the system as a function of the number of particles N and of the volume V. Show that E_0 is extensive.

5 points

Gas of Molecules

Consider a gas of N molecules, each of with mass m, in a volume V and at a temperature T.

If the density of the molecules $n = N/V$ is sufficiently low we can neglect quantum effects coming from the overlapping of wavefunctions of two distinct molecules. This means that the translational degrees of freedom can be treated classically and the partition function of the gas becomes:

$$Z_N = \frac{V^N}{N!\lambda_T^{3N}} (Z_{int})^N \tag{1}$$

where Z_{int} is the partition function of each single molecule.

a) For which values of the density is Eq. (1) a good approximation?

We consider an energy spectrum for each molecule given by $E_n = \alpha n$ with $n = 0, 1, \ldots \infty$ and suppose that each level is $2n + 1$ degenerate.

b) Calculate Z_{int} in absence of magnetic field h. Derive also the energy and specific heat. How does the *total* specific heat of N molecules behave at high temperatures?

5 points

Bose-Einstein condensation

Consider a gas of non-interacting identical bosons in a cubic box of size L. Let us suppose that each Boson has an energy:

$$\varepsilon = \alpha \left|\vec{p}\right|^s$$

where $\alpha, s > 0$.

a) From the quantization condition on momentum (using periodic boundary conditions) determine the density of states $g(\varepsilon)$.

b) Express the total number of particle per unit of volume as an integral over ϵ and determine for which values of s one has Bose-Einstein condensation.

c) In the case that Bose-Einstein condensation occurs show that below the condensation temperature the pressure becomes independent of the volume.