Examen Differentiaalvergelijkingen

Voormiddag 14/01/2019

Woordje uitleg

Dit is pas na de examenperiode aangemaakt door yours truly die per ongeluk twee examenvragen niet had afgegeven. Dit zijn de twee vragen van het deel van professor Lapenta (Hoofdstukken 7 t.e.m. 13, waarvan 13 bij ons geen examenstof was). Dit deel is volledig schriftelijk.

De andere vragen herinner ik me helaas niet in detail, waarvoor mijn excuses.

Vraag 3: Telegrapher Equation

The Voltage and current in an electrical transmission line are governed by the *telegraph equation*:

$$\frac{\partial V}{\partial x} = -L\frac{\partial I}{\partial t}$$
$$\frac{\partial I}{\partial x} = -C\frac{\partial V}{\partial t}$$

where L is the inductance and C the capacitance of the line.

Deelvraag 1

The system of two first order equations together form a second order partial differential equation. Identify the type (elliptic, parabolic or hyperbolic).

Deelvraag 2

Using the method of Fourier transformation for an infinite domain in x, consider the solutions of the temporal type

$$V(x,t) = Re\{V(x)\cdot e^{i\omega t}\}$$
$$I(x,t) = Re\{I(x)\cdot e^{i\omega t}\}$$

and derive the relationship between ω and the Fourier variable k.

Deelvraag 3

Given now an initial condition such that $V(x,t=0) = \cos(x)$ and $I(x,t=0) = \sin(x)$

Vraag 4: Hermitian property

Consider the following differential operator:

$$(Xf)(x) = -\frac{1}{\cosh(x)}\frac{d}{dx}\left(e^{x^2}\frac{df}{dx}\right) + \frac{\tan(x)}{\cosh(x)}f$$

in the interval $-1 < x < 1$ subject to the boundary conditions

$$f(-1) = 0 \qquad\qquad f'(1) = 0 \tag{1}$$

Answer the following questions:

1. Is the operator Hermitian?
2. Are the eigenvalues real?
3. Will the eigenfunctions form a complete basis?
4. Is the Sturm-Liouville problem regular or singular?