Mathematical Introduction to Fluid Dynamics

25/08/2021

Exercises

The exercise exam counts for 10/20 marks. The scores of the two best exercises are averaged to form the final score for this part of the exam.

1. At a previous conference I received a coffee mug with many different quantities on them, similar to the Reynolds number. One specific mentioned number is the *Ekman number* Ek, which is given by
$$\mathrm{Ek} = \frac{\nu}{\Omega L^2}.$$
With the hint that this is a quantity that is important in oceanography and rotating astrophysical bodies, come up with a physical interpretation and the significance of the Ekman number. Be sure to include formulas to derive the Ekman number in a proposed setting! What happens in the limit of $\mathrm{Ek} \to 0$ and $\mathrm{Ek} \to \infty$?
(HINT: The Coriolis force $\vec{F}_c$ is given by $\vec{F}_c = -2m(\vec{\Omega} \times \vec{v})$.)

2. To describe the flow around a corner, the complex transformation $s = h(z) = z^n$ is often used.
   (a) Find what the image under this transformation is of a half-line starting at $z = 0$.
   (b) Find the potential in a corner of $\pi/2$, coming from infinity at a uniform speed. Make a sketch of the streamlines.
   (c) Find the stagnation point in this flow.

3. In the lecture, we have used the linear density-velocity relationship
$$v = v_{max}(1 - \rho/\rho_{\max})$$
in traffic flow theory. This implies that drivers use their car to the maximum speed that feels comfortable. In practice, there are a lot of speed restrictions in place. This has as consequence that the density-velocity relationship reaches a flat top: for low densities, the speed is fixed to the speed limit V, rather than increasing linearly with density.
   (a) Compute the flux in such a traffic regime.
   (b) Obviously, for $V > v_{max}$, the maximum flux $q^\star$ does not depend on V. For which speed limits V does the maximum flux $q^\star$ depend on this speed limit?
   (c) Compute the linear traffic wave speed in this traffic flow regime.