Radiative Processes in Astronomy

Monday 17 August 2020

The exam is divided in 4 questions.

You are allowed to use the course lecture-notes, and a calculator. It is NOT allowed to use a mobile phone as calculator, nor any other mobile device. It is also NOT allowed to use any notes from solutions to exercises done in class or by yourself, nor any of the pdf-slide lectures.

Please write your name on each paper that you use and on the papers with questions. In order to get points on the written questions (1,2,3), you must show clearly your full work. Make sure that your hand-writing is well readable and that you follow a logical structure for your calculations and when answering the questions.

– If your first solution to a problem looks "messy", we advice you to simply do and hand in a re-writing of your solution using a new, fresh set of papers.

NOTE-1: Partial credit will be given in exercises (1)-(3), so, for example, if you are not able to find reasonable numerical values where such are asked for, still provide your full calculations and a discussion about what you think might have gone wrong.

NOTE-2: For the exercises in the exam, you may need the hydrogen atom mass m_H and electron mass m_e. In CGS units they are $m_H = 1.6737 \times 10^{-24}$ g and $m_e = 9.1094 \times 10^{-28}$ g. There is a typo in the table in the lecture-notes, where they unfortunately have been exchanged (thanks a lot for pointing this out!). If you have further questions, just ask Jon.

Good luck !

1. Select **two** of the following key concepts from the course, and carefully write down a description and explanation of them using your own words. Discuss relevant astrophysical applications [7 points] :

   - Elastic scattering as a classical oscillator, including the three regimes and relevant applications
   - Wien's displacement law and how it can be used to roughly estimate surface temperatures of stars
   - Thermal Bremsstrahlung including astrophysical applications
   - Atomic transitions of hydrogen (physics origin, spectral domains, astrophysical applications)
   - Basic radiative transfer through homogeneous slab

2. Line broadening and line formation [3 points] :

   (a) Let us consider the hydrogen Lyman α spectral line, formed in a neutral pure hydrogen interstellar gas cloud of $T = 200$ K. For this situation, compute what mass density (in g/cm^3 or kg/m^3) the cloud must have in order for collisional line broadening to be equally important as thermal Doppler line broadening at FWHM? What does your calculation tell you about the relative importance of collisional and Doppler line broadening for line formation in such interstellar gas?

       HINT: Collision line broadening has a Lorenz profile with damping parameter γ as given in lecture notes. To estimate the mean time τ between collisions you can here assume $\ell = v_{\rm th}\tau$, and estimate the collisional mean-free-path ℓ by simply assuming the geometrical cross-section of a Bohr atom.

3. Ionization and radiation energy density in the Universe [6 points] :

   (a) For a pure hydrogen gas in LTE with mass density $\rho = 10^{-9}\,\mathrm{g/cm^3}$ and temperature $T = 5800$ K, compute the fraction of ionized to total number of hydrogen atoms. Are the hydrogen atoms mostly neutral or ionized? What astrophysical object might this be a reasonable approximation for? In Ch. 8.1 of the lecture-notes, we compute the ionization state of a hydrogen Universe as a whole, and find that at $T = 5800$ K the Universe is almost fully ionized. Does this result differ from the one you have found in your calculation here? If so, why? [3 points]

       NOTE: For the hydrogen partition functions, you can assume $U_{HII}/U_{HI} = 1/2$.

       MATH: Solution to equation $ax^2 + bx + c = 0$ is
       $x = (-b \pm \sqrt{b^2 - 4ac})/(2a)$.

   (b) Using measurements of the CMB radiation temperature $T_0 = 3\,\mathrm{K}$ and mass-density $\rho_0 \approx 3 \times 10^{-31}\,\mathrm{g/cm^3}$ in the present-day Universe, estimate the redshift z at which the radiation energy density E_R and the rest-mass energy density $E_M = \rho c^2$ of the Universe were equal. For this you may assume radiation in thermal equilibrium, mass conservation in a sphere with scale factor R, and that the following relations hold: $R \propto (1+z)^{-1}$ and $T \propto 1+z$. From your calculation, determine which one of the radiation density E_R and matter density E_M is highest in the Universe i) today and ii) at very high redshifts near the Big-Bang. [3 points]

       NOTE: At present time $z = 0$.

4. Quiz [4 points]:

Correct answer: 0.5 points/question. Incorrect answer: 0 points/question.

(a) Assuming pure extinction of a light-source, at optical depth 3
1) less than 10 % of the emitted photons escape
2) between 10-20 % of the emitted photons escape
3) no photons escape

(b) For radiation transport through two homogeneous, optically thick slabs, 1 and 2, in LTE with zero incoming intensities, and of temperatures $T_1 < T_2$, the outgoing (emergent) intensities I_1 and I_2 will have:
1) $I_1 = I_2$
2) $I_1 > I_2$
3) $I_1 < I_2$

(c) The radiation pressure of a black-body depends on its:
1) density
2) temperature
3) density and temperature

(d) For a black-body radiator with $T = 10^5 K$, we can apply the Wien approximation when analyzing its infrared radiation
1) Yes, if $\rho > 10^{-9}$ g/cm^3
2) Yes
3) No

(e) For thermal bremsstrahlung with $h\nu/(kT) \ll 1$, the optical depth in a homogeneous medium depends on

1) ν^0
2) ν^2
3) ν^{-2}

(f) In the radiative diffusion approximation, the following relations between the intensity moments J, H, K always hold:

1) $J/K = 3$ and $H/K = 0$
2) $H/K = 0$ but not necessarily $J/K = 3$
3) $J/K = 3$ but not necessarily $H/K = 0$

(g) Scattering of photons generally tend to drive a gas

1) into TE
2) out of TE but into LTE
3) out of TE and out of LTE

(h) The effective temperature of an astrophysical object is really a proxy-quantity for its

1) intensity
2) radiative flux
3) energy density