Stellar Structure and Evolution

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We could bring the notes Pablo wrote (42 p.) and a calculator. This years, Chapters 10-12 of Conny Aerts' notes needed to be known and couldn't be brought to the exam. Each of the 5 questions on this exam is equally weighted.

Question 1

Part 1) Consider a star with constant density,

$$\rho(m) = \rho_c, \quad m(r) = \frac{4\pi}{3} r^3 \rho_c.$$

Use the equation of hydrostatic equilibrium to obtain the interior pressure of the star as a function of the central pressure P_c and r/R, where R is the total radius of the star. Assume that the pressure at the surface of the star is much smaller than the central pressure.

Part 2) Using the Lagrangian form of the equation of hydrostatic equilibrium,

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4},$$

place a lower bound on the central pressure of the sun by making use of the basic property that anywhere within the stellar interior we have that $r < R_\odot$.

Question 2

The structure of a hypothetical star with constant entropy would be given by

$$\frac{\partial T}{\partial P} = \nabla_{\rm ad} = \frac{P\delta}{T\rho c_p}.$$

This is a good approximation for a fully convective star. Remembering the ideal gas law

$$P = \frac{k_B T}{\mu m_u}\rho$$

and the internal gas energy

$$u = \frac{3k_B T}{2\mu m_u},$$

show that $\nabla_{\rm ad} = 2/5$, and that this implies that the structure of the star can be described using an $n = 3/2$ polytrope. Remember that the specific heat at constant pressure can be computed from

$$c_p = \left(\frac{\partial u}{\partial T}\right)_P + P\left(\frac{\partial v}{\partial T}\right)_P.$$

Question 3

The pressure in a gas with radiation can be divided in its contributions from each component,

$$P = P_{\text{gas}} + P_{\text{rad}}, \quad P_{\text{rad}} = \frac{aT^4}{3}.$$

The equation of hydrostatic equilibrium can then be expressed as

$$\frac{\partial P_{\text{gas}}}{\partial r} + \frac{\partial P_{\text{rad}}}{\partial r} = -\frac{GM\rho}{r^2}.$$

- Using this, find the value of the luminosity for which the radiation pressure would overcome the gravitational force. For this purpose use the expression for the temperature gradient under the assumption that all energy is transported by radiation,

  $$\frac{dT}{dr} = \frac{-3}{16\pi ac}\frac{\kappa\rho l}{r^2T^3}.$$

  This luminosity is known as the Eddington luminosity L_{edd}, and you should find that

  $$L_{\text{edd}} = \frac{4\pi Gmc}{\kappa}.$$

- If you look at the ratio L/L_{edd}, what can you say about this fraction for a higher mass star? Suppose the surface opacity is not changing with mass.
- For a more compact object with the same mass, the surface gravity increases strongly. So, a naive approach would lead you to think L_{edd} will be larger for this compact object. However, L_{edd} is independent of the radius R, explain physically why.

Question 4

Suppose the pressure of a star has again a gas and a radiation component. The ideal gas law then becomes

$$P = \frac{\rho}{\mu}(m_u k_B T) + \frac{aT^4}{3}.$$

We can define

$$\beta = \frac{P_{\text{gas}}}{P} = 1 - \frac{P_{\text{rad}}}{P}.$$

Part 1) Show that we can write

$$P = C\left(\frac{\rho}{\mu}\right)^{4/3}(f(\beta))^{1/3}, \quad \text{where } f(\beta) = \frac{1-\beta}{\beta^4}$$

and C is a combination of universal constants.

Part 2) Before computers were available to solve stellar models, Arthur Eddington performed solutions to the stellar equations by approximating β to be constant along the star's structure. Explain why it is easier to work with a constant β. Don't overthink this, you can explain in one or two sentences.

Part 3) In class, we derived homology relations when no radiation pressure is present. Now, get a homology relation for $f(\beta)$ using the continuity equation, the equation of hydrostatic equilibrium and the ideal gas law including the radiative pressure.

Part 4) For $0 < \beta < 1$, we have that $f(\beta)$ is a monotonically decreasing function. Using this and your homology relation from the previous part, can you say anything about a more massive star having a larger contribution of the radiative pressure to the total pressure?

Question 5

Below, you find an empty HR diagram with a dot representing the TAMS point of a 1 $M_\odot$ star. Draw the evolution of this star on the HR diagram untill a compact object is formed. Indicate with numbers the different stages where different burning and depletion of elements occur. Explain what happens in each phase and give the names of these phases.