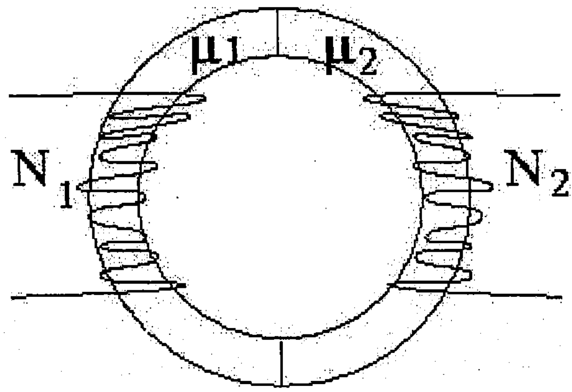


1 Consider a transformer, i.e. a device for raising or lowering the emf of an alternating current source. Specifically, consider a model in which two coils are wrapped around a ferromagnetic, torroidal core as shown in the Figure. The core is made from two different pieces of linear ferromagnetic with high permeability. The primary coil has N_1 turns and is wrapped around the region with permeability μ_1, whereas the secondary coil has N_2 turns and is wrapped around the region with permeability μ_2. Assuming that the current in the primary coil is changing, determine the ratio between the emf in the secondary coil and the (back) emf in the primary coil. Does your answer depend on μ_1, μ_2? Explain why. (Note: field lines always localize inside materials with high permeability.)

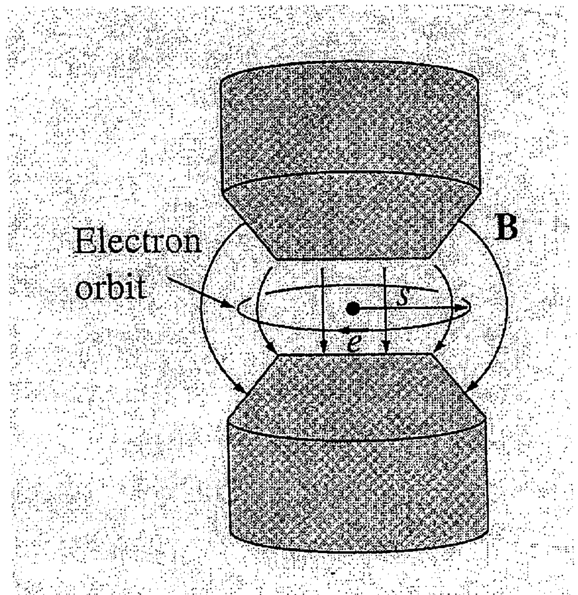


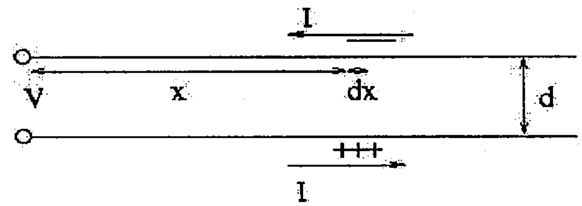
Figure 7.52

2 Griffiths 7.48 Electrons undergoing cyclotron motion can be sped up by increasing the magnetic field; the accompanying electric field will impart tangential acceleration. This is the principle of the betatron. One would like to keep the radius of the orbit constant during the process. Show that this can be achieved by designing a magnet such that the average field over the area of the orbit is twice the field at the circumference (see Figure). Assume the electrons start from rest in zero field, and that the apparatus is symmetric about the center of the orbit. (Assume also that the electron velocity remains well below the speed of light, so that nonrelativistic mechanics applies.) [Hint: differentiate the cyclotron formula $p = qBR$ with respect to time, and use F = ma = qE for a proper component of force and momentum.]

3 Griffiths 7.44 If a magnetic dipole levitating above an infinite superconducting plane is free to rotate, what orientation will it adopt, and how high above the surface will it float? Assume that the mass M and the absolute value of the dipole moment m are given. [Notes: B=0 inside superconductors; make use of the method of images. Make sure to select the orientation that corresponds to a *stable* equilibrium.]

4 Consider the propagation of an electromagnetic wave in a medium where the binding energy of electrons is much smaller than the frequency of the field. In such a case the medium can be considered as consisting of free electrons. This is the case, for example, for high-frequency radiation propagating in ionized gases or metals. Assume that the charge e, the concentration of free electrons n, and the mass m of the free electrons are given. (Assume further that the medium is electrically neutral due to heavy ions that do not contribute to the susceptibility).

- Show that in such a medium the electric field is proportional to the time derivative of the current density J. Determine the proportionality constant between the two.
- Starting from Maxwell's equations, derive the evolution equation (modified wave equation) for the electric field in this medium.
- For a plain monochromatic wave $E(z,t) = E_0 \exp(ikz - i\omega t)$, derive the dispersion relation (i.e. the relation between k and ω).
- Show that for frequencies below the certain value ω_p wave propagation in such a medium is forbidden. Derive the expression for ω_p and the expression for the intensity of the field inside the medium right after its boundary. Describe physically what will happen with a plain wave that is incident onto the boundary from free space at a right angle.



5 Consider a transmission line made of two infinitely long parallel wires as shown in the Figure. A time-varying voltage applied at the end of this system will excite a wave of current and voltage propagating down this transmission line. Assume that the resistance of the wires per unit length R is given.

- Consider how the charge, current and voltage change at a position x over the distance dx. Derive the relations between the linear charge density (charge per unit length) $\lambda(x,t)$ and the current $I(x,t)$ and between the voltage $V(x,t)$ and a magnetic flux per unit length in the presence of resistance.

- Introduce a capacitance per unit length C and an inductance per unit length L as follows: $\lambda = CV$, $\Phi = LI$. Derive the evolution equations (partial differential equations) for I and V.

- Derive the dispersion relation (i.e. the relation between k and ω) for a plain monochromatic wave propagating along the transmission line.

- Assuming that losses are small, find the dependance of the intensity on x and determine the characteristic propagation distance of the wave. The latter is defined as the distance over which the intensity of the wave is attenuated by exp(1). Provide the physical interpretation for your result.