Exam quantum mechanics

June 2016

You can use the book. Be clear, what is not clear cannot be corrected. You can answer in Dutch, French or English. If you want a Dutch translation of the exam questions, feel free to ask. Good luck!

Short questions [11ptn].

- Consider a gas of free particles at zero temperature. Derive the Fermi energy and the pressure in case the particles have spin 5/2. What is Fermi pressure when the spin is 5? [4 pts]
- Consider a system with N degrees of freedom such that its wave function is an element of an N-dimensional (complex) vectorspace. Now consider the collection of M such system. What is the dimension of the (complex) vector space describing this multi-particle system? [2 pts]
- Consider again a spin 1/2 particle whose wave-function is

$$|\Psi\rangle = \begin{pmatrix} a(\vec{r},t) \\ b(\vec{r},t) \end{pmatrix} \tag{1}$$

  with a and b certain specified functions. Now perform the following coordinate change $x' = y, y' = -x, z' = z$. How does the wavefunction look like in the new coordinate system? [2 pts]
- Prove that the classical equation $\frac{d}{dt}\vec{L} = \vec{r} \times \vec{F}$ can be obtained in quantum mechanics [3pts].

Isotropic harmonic oscillator [9ptn]

Consider the isotropic harmonic osscillator defined by the potential

$$V(\vec{r}) = \tfrac{1}{2}kr^2\,. \tag{2}$$

The solutions can be found in the book and you can use those results from here on. You will see in the book that the solutions can be written in spherical coordinates or in Cartesian coordinates. The solutions in Cartesian coordinates are labbeled by the quantum numbers n_x, n_y, n_z (where $n = n_x + n_y + n_z$ determines the total energy) and written as $\Psi_{n_x n_y n_z}$. In spherical coordinates the quantum numbers are n, l, m and states are written as Ψ_{nlm}. In this exercise we care about the relation between the two ways of labeling the solutions but we restrict ourselves to the first excited state ($n = 1$).

- How do you compute the degeneracy of the $n = 1$ state in both coordinates? [2pt]
- Compute the action of L_z on $\Psi = c_x\Psi_{100} + c_y\Psi_{010} + c_z\Psi_{001}$ with c_x, c_y, c_z complex numbers. [4ptn]
- What is the relation between c_x, c_y and c_z in order for Ψ to be proportional to $\Psi_{n=1,l=1,m=1}$? [4ptn]