Problem 1 (oral)

Let $X = l^2(\mathbb{N})$ and let $\chi_n = \mathbb{1}_{\{1,2,\ldots,n\}}$. Note that $X = (l^\infty(\mathbb{N}))^\star$. Does χ_n converge in the weak * topology? What is its limit? Does it converge in the weak topology?

Problem 2

Let $H = L^2[0,1]$ and T an integral operator with kernel $K(x,y) := x + y$. Is T self adjoint? Is it compact? What is its spectrum? What are the eigenvectors to its nonzero eigenvalues?
Compute the dimension of the kernel.

Problem 3

Let $H = l^2(\mathbb{N})$ and let $V_n x(k) = x(n+k)$. Show it converges to zero in the strong operator topology but $V_n^\star$ does not converge in the strong operator topology.
Show that if W_n are isometries and $W_n \to W$ in the strong operator topology, W is an isometry.

Problem 4

Let $X = C^1[0,1], Y = C[0,1]$ equiped with the supremum norm. Let $T : X \to Y : f \to f'$. Show that the graph of T is closed yet T is not continuous.