Relativity

(27/08/2013 (14u00-18u00))

1 Using Scharzschild coordinates, show that *every* timelike curve in region II of the Kruskal manifold intersects the singularity at $r = 0$ within a proper time no greater than πGM. For what curves is this bound attaind?

2 (a) Consider a initial Schwarzschild black hole of mass M that evolves to form two well-seperated approximately Schwarzschild black holes at rest. Use conservation of eneryg and the second law of black hole mechanics to show that this cannot happen. (This is a special case of a completely general result that black holes cannot bifurcate.)

(b) Suppose two widely sepreated Kerr black holes with parameters (M_1, J_1) and (M_2, J_2) are initially at rest in an axisymmetric configuration, i.e. their rotation axes are alignes along the direction of their seperation. Assume that these vlack holes fall together and coalesce into a single black hole. Since angular momentum cannot be radiated away in an axisymmetric space-time, the final black hole wil have an angular momentum $J_1 + J_2$. Derive an upper limit for the energy radiated away in this process. Is this upper limit larger when J_1 and J_2 are antiparallel rather than parallel?
Hint: the solutions of $2x^2(1+\sqrt{1-a/x^4}) = b$ *with* a *and* b *constants are* $x = \pm\sqrt{a+b^2}/\sqrt{2b}$.

(c) Show that the area of the event horizon of a Kerr-Newman black hole with rotation parameter a and electric charge Q is

$$A = 8\pi G^2\left(M^2 - \frac{Q^2}{2} + \sqrt{M^4 - (J/G)^2 - M^2Q^2}\right).$$

3 The generalization of the Schwarzschild solution to spacetimes with a cosmological constant Λ is given by the following metric (in units where $c = G = 1$),

$$\mathrm{d}s^2 = -\left[1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}\right]\mathrm{d}t^2 + \left[1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}\right]^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2.$$

(a) Discuss the causal structure of the Schwarzschild-de Sitter spacetime given by with $\Lambda > 0$, e.g. by drawing the Kruskal diagram.

(b) Discuss the behavior of geodesics of massive particles in these spacetimes, in particular how a nonzero cosmological constant (either positive of negative) modifies the bound orbits of the Schwarzschild geometry.

4 (a) Consider the Robertson-Walker universe that best fits the current observations, with density parameters $\Omega_{R0} = 10^{-4}$, $\Omega_{M0} = 0.3$ and $\Omega_{\Lambda 0} = 0.7$.
Sketch the behavior of the three Ω's as a function of the scale factor a on a log scale from $a = 10^{-35}$ to $a = 10^{35}$. Indicate the Planck time, nucleosynthesis and today.

(b) Sketch the evolution of the scale factor in this universe. Indicate the period during which large-scale structures such as galaxies form. What would the universe have been like if Λ had been significantly larger?

(c) Consider slow-roll scalar field driven inflation in a scalar potential $V = \lambda\phi^n$ for $n \geq 2$. Find the slow-roll parameters ϵ and η. Assuming inflation ends when $\rho + 3p = 0$ and assuming slow-roll holds all the way to the end of inflation (i.e. $3H\dot{\phi} \approx -V_{,\phi}$), calculate the number of e-foldings N as a function of the value ϕ_i of the scalar field at the start of inflation.