Exam Particle Physics

June 2013

You are allowed to use the formula sheet. On that sheet we take $\hbar = c = 1$, which you can use as well.

Question 1: muon decay

- Prove that the scattering amplitude for μ/e^- scattering is equal to the scattering amplitude of μ/e^+ scattering by using Casimirs trick.[Advice: it is not necessary to compute the whole amplitude, just compute enough to show that they are equal]
- It is possible that you use the following trace identity for answering the above question:

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\sigma] = 4(g^{\mu\nu}g^{\lambda\sigma} - g^{\mu\lambda}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\lambda})\,.$$

  Prove this.
- Write the amplitude for e^-, e^- and e^+, e^- scattering in spinor form, you do not need to work it out. Is this analogues to the above muon/electron versus anti-muon/electron scattering in the sense that the cross sections of both electron/electron and electron/positron scattering are equal? [Answer yes or no, with a very brief comment on why so.]

Question 2: Dirac fermions

- The Dirac Lagrangian density reads

$$\mathcal{L} = i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - m\bar{\Psi}\Psi$$

  Is this Lagrangian density real? If not, then discuss whether this is a problem, since actions in field theory are defined to be real.
- Discuss how the principle of U(1) gauge invariance dictates the coupling between a Dirac fermion Ψ and a Maxwell vector field A. (Write the action, explain how you get it.)
- The action you obtained defines a 4-vector current j^μ that couples to the vector. What is this current and check whether the current is conserved.