Fundamentals of financial mathematics

27/01/2024

1 Question 1

1. Proof and give the put call parity under the no arbitrage condition in the one price world. Assume that there are no dividends.
2. Give (not proof) the put call parity in case there are non-negative dividends.
3. Give the put call parity for the two price world. (no proof) (no dividends)
4. Give and prove the calendar spread inequality in a one price setting for the European Call (assuming no dividends).

2 Question 2

1. Explain in detail the pricing of American options under a 3-step binomial tree model in general. Give general formulas (assume a general S_0, r, T, and payoff function). Assume $q = 0$ (no dividends). Draw the tree and show the details of price calculation at each step.
2. Illustrate it by the pricing of a digital American option paying 0 if the stock price at exercise is lower than the strike K and 1 otherwise under a setting with $S_0 = 100; K = 105; r = q = 0$; and $u - 1 = 1 - d = 0,1$.

3 Question 3

1. What is a complete model? Give briefly a definition/explanation of the concept.
2. Discuss in a finite discrete market model the relationship between an equivalent Martingale measure and completness of the market model. Give briefly a definition/explanation of these concepts and state the main theorems. (no proofs)
3. Explain the difference between the real world and the risk neutral world.

4 Question 4

1. Relate the following Black-Scholes PDE for the price O of an option under the Black-Scholes model $\frac{\partial O}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 O}{\partial S^2} + rS\frac{\partial O}{\partial S} - rO = 0$ to the statement "the total change in the value of a delta hedged portfolio is equal to 0 on average".