Linear Optimisation Study Set 1

1. Candy Production Blending Problems

Willy Wonka's Candy Company Produces three types of Candy

1.Wonka Bars
2.Bottle Caps
3.Giant Sweet Tarts

In order to produce the different type of candies, Willy can run three different production processes as described below. Each process involves blending different types of sugars in the Magical Factories Mixer.

Process 1:
Running Process 1 for one hour:
Costs: $5
Requires: Two barrels of sugar type A and three barrels of sugar type B
Output: Two Wonka Bars and one packet of Bottle Caps

Process 2:
Running Process 2 for one hour:
Costs: $4
Requires: One barrel of sugar type A and three barrels of sugar type B
Output: Three packets of Bottle Caps

Process 3:
Running Process 1 for one hour:
Costs: $1
Requires: Two barrels of sugar type B and three packets of Bottle Caps
Output: Two Wonka Bars and one packet of Giant Sweet Tarts

Incidentally, the barrels are really tiny, and the candy is very sweet.

Each week we can purchase:

- 200 barrels of sugar type A at $2 Per Barrel
- 300 barrels of sugar type B at $3 Per Barrel

Assume that they can sell everything that they can produce.

- Wonka Bars are sold at $9 per bar.
- Bottle Caps are sold at $10 per packet.
- Giant Sweet Tarts are sold at $24 per packet.

Assume that 100 hours of mixing time are available.

a. Formulate an LP whose solution will maximize Willy Wonka's Profits.

b. Assume that instead of having 200 barrels of sugar type A and 300 barrels of sugar type B available that you can order a total of 500 Barrels. Show how to modify your LP formulation in Part A to account for this revised problem.

c. Suppose that instead of selling the three candies separately, they can only be sold as part of a box consisting of one Wonka Bar, two packets of Bottle Caps, and one pack of Giant Sweet Tarts. Each Wonka Box sells for $54. Modify your LP formulation in part A to model this new scenario.

2. A company wishes to plan its production of two items with seasonal demands over a 12 month period. The monthly demand of item 1 is 100,000 units during the months of October, November, and December; 10,000 units during the months of January, February, March and April; and 30,000 units during the remaining months. The demand of item 2 is 50,000 during the months of October through February and 15,000 during the remaining months. Suppose that the unit product cost of items 1 and 2 is $5.00 and $8.00, respectively, provided that these were manufactured prior to June. After June, the unit costs are reduced to $4.50 and $7.00 because of the installation of an improved manufacturing system. The total units of items 1 and 2 that can be manufactured during any particular month cannot exceed 120,000 for Jan-Sept, and 150,000 for Oct-Dec. Furthermore, each unit of item 1 occupies 2 cubic-feet and each unit of item 2 occupies 4 cubic-feet of inventory. Suppose that the maximum inventory space allocated to these items is 150,000 cubic-feet and that the holding cost per cubic-foot during any month (incurred at the end of the month) is $0.10. Formulate the production planning problem so that the total production and inventory costs are minimized.

3. Suppose that you are working at the production planning department of a company and you have the demand forecasts for the next 4 months. We denote the demand in month t with dt. We have d1 = 4300, d2 = 4100, d3 = 3400 and d4 = 1800.
Your company has 40 workers and each worker works for 140 hours of regular time during a month.
To adjust to the fluctuations in the demand values, the management can use any combination of the following strategies:
1) Use overtime labor
2) Hold inventory
Each of these strategies has limitations. In particular, each worker can work at most 40 hours overtime during each month. Since the company has limited storage space, no more than 800 products can be stored.
Each product requires 1.5 hours of labor. The hourly cost of overtime labor is 30 euros.
For each product stored in inventory at the end of each month, 0.5 euros of holding cost is charged. In addition, there is spoilage in inventory. Of the amount stored at the end of a month, 10% must be discarded in the following month due to spoilage.

At the beginning of the planning horizon, there are no products in inventory.
Formulate a linear program to minimize the total cost while meeting the demand on time.

4. Consider the problem of locating a new machine to an existing layout consisting of four machines. These machines are located at the following $x1$ and $x2$ coordinates: (3, 1), (0, -3), (-2, 2), and (1, 4). Let the coordinates of the new machine be $(x1, x2)$. Formulate the problem of finding an optimal location as a linear program for each of the following cases:
a. The sum of the distances from the new machine to the four machines is minimized. Use the street distance: for example, the distance from $(x1, x2)$ to the first machine located at (3, 1) is $|x1-3|+|x2-1|$.
b. Because of various amounts of flow between the new machine and the existing machines, reformulate the problem where the sum of the weighted distances is minimized, where the weights corresponding to the four machines are 6, 4, 7, and 2, respectively.
c. Suppose that the new machine must be located so that its distance from the first machine does not exceed 2. Formulate the problem with this added restriction.
d. Now suppose that we would like to minimize the largest of the distances from the new machine to the four machines. Formulate this new variant as an LP.

5 Consider the linear programming problem

$$\begin{aligned} &\max\ x1 + x2 \\ &\text{s.t.} \quad \mathbf{s}x1 + x2 \le \mathbf{t} \\ &\qquad\quad x1, x2 \ge 0. \end{aligned}$$

Find some values for **s** and **t** such that this linear program

a. has a unique optimal solution.
b. has multiple optimal solutions.
c. is infeasible,
d. is unbounded.

Illustrate each case with a graph.