# Linear Optimisation

More modeling exercises

# A multi-plant model from Williams (2013)

A company operates two factories, A and B.

- Each factory
  - makes two products: *standard* and *deluxe*.
  - uses two processes: grinding and polishing

  The grinding and polishing times in hours for a unit of each type of product in each factory are as follows:

| | Factory A | | Factory B | |
|---|---|---|---|---|
| | Standard | Deluxe | Standard | Deluxe |
| Grinding | 4 | 2 | 5 | 3 |
| Polishing | 2 | 5 | 5 | 6 |

- Factory A has a grinding capacity of 80 hours and polishing capacity of 60 hours per week.
- Factory B has a grinding capacity of 60 hours and polishing capacity of 75 hours per week.

# A multi-plant model

- A unit of *standard* gives a profit contribution of $10, while a unit of *deluxe* gives a profit contribution of $15.
- Each unit of each product uses 4 kilograms of raw material.
- The company has 120 kilograms of raw material available per week. Assume that
  - Factory A is allocated 75 kilograms of raw material per week
  - Factory B is allocated 45 kilograms of raw material per week
- For each factory, formulate an LP whose solution maximizes its profit.

# A multi-plant model

- Now suppose that a company model is built in order to maximize total profit.
- There will be a single raw material constraint limiting the company to 120 kilograms per week. No longer allocate 75 kilograms of raw material to A and 45 kilograms to B.
- What would be the new model?
- Can you compare the optimal value of the company model with those of the factory models?

# Exercise 1 from Bazaraa et al. (2011)

- An agricultural mill manufactures feed for cattle, sheep and chickens by mixing corn, limestone, soybeans, and fish meal.
- These ingredients contain vitamins, protein, calcium and crude fat. The content of the nutrients in each kilogram of the ingredients are as follows:

| | Nutrient | | | |
|---|---|---|---|---|
| Ingredient | Vitamins | Protein | Calcium | Crude Fat |
| Corn | 8 | 10 | 6 | 8 |
| Limestone | 6 | 5 | 10 | 6 |
| Soybeans | 10 | 12 | 6 | 6 |
| Fish meal | 4 | 8 | 6 | 9 |

- The mill is contracted to produce 10, 6 and 8 tons of cattle feed, sheep feed and chicken feed, respectively.
- A limited amount of the ingredients is available:
  - 6 tons of corn whose price per kilogram is \$0.20
  - 10 tons of limestone whose price per kilogram is \$0.12
  - 4 tons of soybeans whose price per kilogram is \$0.24
  - 5 tons of fish meal whose price per kilogram is \$0.12
- The minimal and maximal units of the nutrients that are permitted for a kilogram are as follows:

| | Nutrient | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Vitamins | | Protein | | Calcium | | Crude Fat | |
| Product | min | max | min | max | min | max | min | max |
| Cattle feed | 6 | $\infty$ | 6 | $\infty$ | 7 | $\infty$ | 4 | 8 |
| Sheep feed | 6 | $\infty$ | 6 | $\infty$ | 6 | $\infty$ | 4 | 6 |
| Chicken feed | 4 | 6 | 6 | $\infty$ | 6 | $\infty$ | 4 | 6 |

- Formulate this problem so that the total cost is minimized.

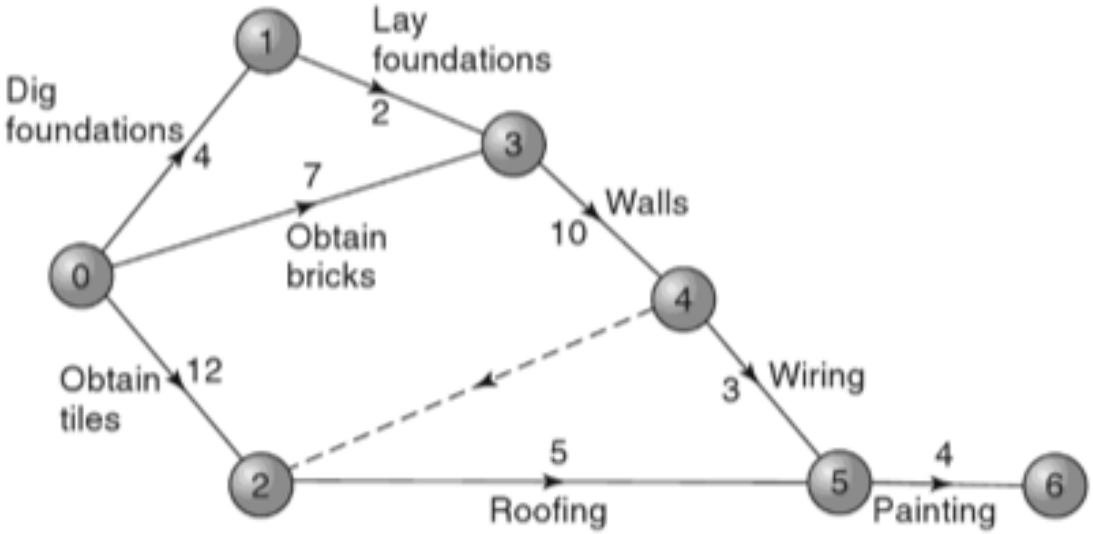
# Exercise 3 from Bazaraa et al. 2011

- The quality of air in an industrial region largely depends on the emission from $n$ plants.
- Each plant $j$ can use $m$ different types of fuel.
- Plant $j$ needs $d_j$ thermal units of total energy per day.
- Each fuel type $i$ costs $c_i$ dollars per ton and generates $a_{ij}$ thermal units of energy at plant $j$.
- $e_{ij}$ is the emission per ton of fuel type $i$ at plant $j$.
- The level of air pollution in the region is not to exceed $b$ micrograms per cubic meter.
- $\gamma_j$ is the meteorological parameter relating emissions at plant $j$ to micrograms per cubic meter.

**a)** Formulate the problem of determining the mix of fuels to be used at each plant.

**b)** How could you ensure equity among the plants?

# Critical path from Williams (2013)




The network in the figure represents a project of building a house. Each arc represents some activity forming part of the project. Durations of the activities are attached to corresponding arcs. Before an activity on arc $i - j$ can start, all activities on arcs coming into node $i$ must be finished. The arc 4-2 marked with a dashed line is a dummy activity having no duration. Its only purpose is to prevent activity 2-5 starting before activity 3-4 has finished.
What is the earliest time the project can be finished?