Managerial economics
Exercises

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Contents

1 Consumer choice

1.1 Problem

Utility function of consumer: $U(X, Y) = AX^\alpha Y^\beta$.
(A, α, β parameters, X = amount of goods x, Y = amount of goods y)
Budget constraint of consumer: $P_X X + P_Y Y = I$.
(I = income, P_i is price of good i)

1. Write down her constrained optimization problem.
2. Derive the first order condition(s).
3. Solve for optimal consumption.
4. How does consumption of Y change if P_X changes?

1.2 Solution

1.

$$Y = \frac{I - P_X X}{P_Y} \Rightarrow U(X) = AX^\alpha \left(\frac{I - P_X X}{PY}\right)^\beta$$

2.

$$d = AX^\alpha Y^\beta + \lambda (I - P_X X - P_Y Y)$$

First order conditions:

1. $\frac{\partial d}{\partial X} = \alpha A X^{\alpha-1} Y^\beta - \lambda P_X = 0$
2. $\frac{\partial d}{\partial Y} = \beta A X^\alpha Y^{\beta-1} - \lambda P_Y = 0$
3. $\frac{\partial d}{\partial \lambda} = I - P_X X - P_Y Y = 0$

3. Extract λ from second conditon:

$$\beta A X^\alpha Y^{\beta-1} - \lambda P_Y = 0 \Rightarrow \lambda = \frac{\beta A X^\alpha Y^{\beta-1}}{P_Y}$$

Extract Y from third condition:

$$I - P_X X - P_Y Y = 0 \Rightarrow Y = \frac{I - P_X X}{P_Y}$$

Fill in λ and Y in first condition:

$$\begin{aligned}
& \alpha A X^{\alpha-1} Y^\beta - \lambda P_X = 0 \\
\Rightarrow\quad & \alpha A X^{\alpha-1} Y^\beta - \frac{\beta A X^\alpha Y^{\beta-1}}{P_Y} P_X = 0 \\
\Rightarrow\quad & \alpha Y = \beta X \frac{P_X}{P_Y} \\
\Rightarrow\quad & \alpha \frac{I - P_X X}{P_Y} = \beta X \frac{P_X}{P_Y} \\
\Rightarrow\quad & \alpha I = (\alpha + \beta) X P_X
\end{aligned}$$

Optimal consumption:

$$X = \frac{\alpha I}{(\alpha + \beta) P_X}$$

Same reasoning for Y gives:

$$Y = \frac{\beta I}{(\alpha + \beta) P_Y}$$

4. Consumption of Y doesn't change.

2 Firm supply decision

2.1 Problem

You can sell as much as you want at the competitive price P.
Cost of producing amount Q: $TC(Q) = F + \omega Q + \delta Q^2$.
$(F \geq 0, \omega > 0, \delta > 0)$

1. What is your average cost of production if you produce $\hat{Q}$ units?

2. What is your marginal cost of production if you produce $\hat{Q}$ units?

3. What are your profits π at an arbitrary level of production Q?

4. At what level of production are average costs minimized?

5. Under what conditions would you produce in the long run?

6. Under what conditions would you produce in the short run?

2.2 Solution

1.

$$AC(\hat{Q}) = \frac{TC(\hat{Q})}{\hat{Q}} = \frac{F}{\hat{Q}} + \omega + \delta\hat{Q}$$

2.

$$MC(\hat{Q}) = \frac{d}{dQ}TC(\hat{Q}) = \omega + 2\delta\hat{Q}$$

3.

$$\pi = PQ - (F + \omega Q + \delta Q^2)$$

4. $AC(Q)$ is minimized if $\frac{d}{dQ}AC(Q) = 0$.

$$\begin{aligned} &\Rightarrow \quad \frac{-F}{Q^2} + \delta = 0 \\ &\Rightarrow \quad Q = \sqrt{\frac{F}{\delta}} \end{aligned}$$

5.

$$P \geq AVC(Q) \Rightarrow P \geq \omega + \delta\sqrt{\frac{F}{\delta}} = \omega + \sqrt{\delta F}$$

6.

$$P \geq AC(Q) \Rightarrow P \geq F\sqrt{\frac{\delta}{F}} + \omega + \delta\sqrt{\frac{F}{\delta}} = 2\sqrt{\delta F} + \omega$$

3 Cost accounting

3.1 Problem

Acounting statement:

	A	B	Total
Revenue	1000	2000	3000
Variable cost	200	1500	1700
Gross profit	800	500	1300
Fixed cost			1000
Profit			300

To find out what is profitable and what is not, you need to assign fixed costs to your products.

1. by using the share of sales as the criterion
2. by using the share of Gross profits as the criterion

Which product line(s) should be (dis)continued using either of the criteria?

3.2 Solution

1.

	A	B
FC	333.33	666.67
Profit	466.67	-166.67

Line A should be continued. Line B should be discontinued.
BUT Next year, B is closed, and $FC = 1000$. Then, $\pi_A = 1000 - 200 - 1000 = -200$. Thus, both lines should be continued.

2.

	A	B
FC	615.38	384.62
Profit	184.62	115.38

Lines A and B should be continued.

4 Monopoly pricing 1

4.1 Problem

Inverse demand: $P = 13 - Q$ (P = price, Q = quantity)
Constant marginal cost: $MC = 1$
Fixed cost of production: F

1. What is your profit function?
2. What is your first order condition?
3. What is your optimal quantity?
4. What is your optimal price and profit?
5. Under what conditions do you want to be in business?
6. How would you answers change if the inverse demand was $P = 18 - Q$?

4.2 Solution

1.

$$\pi(Q) = Q(13 - Q) - (Q + F) = -Q^2 + 12Q - F$$

2.

$$\frac{\partial \pi}{\partial Q} = -2Q + 12 = 0$$

3.

$$Q^* = 6$$

4.

$$P^* = 13 - Q^* = 7$$

$$\pi(Q^*) = -36 + 72 - F = 36 - F$$

5.

$$F \leq 36$$

6.

$$\pi(Q) = -Q^2 + 17Q - F$$

$$\frac{\partial \pi}{\partial Q} = -2Q + 17 = 0$$

$$Q^* = 8.5, P^* = 9.5, \pi(Q^*) = -72.25 + 144.5 - F = 72.25 - F$$

$$F \leq 72.25$$

5 Monopoly pricing 2

5.1 Problem

Inverse demand function: $P = 1060 - 8Q$.
Constant marginal cost: $MC = 100$.
No fixed cost.

1. What is the profit-maximizing quantity you would set?
2. Imagine you can obtain perfect information about the market in exchange for a fee and you could use it in any way you want. How much would you be willing to pay for perfect information?

5.2 Solution

1. Profit:

$$\begin{aligned} \pi &= PQ - 100Q \\ &= (1060 - 8Q - 100)Q \\ &= -8Q^2 + 960Q \end{aligned}$$

First order condition:

$$-16Q + 960 = 0$$

Profit-maximizing quantity:

$$Q^* = 60$$

2. If you have perfect information, you can perfectly price discriminate, thus have a profit equal to the consumer surplus under perfect competition. (Tip: make a drawing.)

$$Q^*_{PPD} = \frac{1060 - 100}{8} = 120$$

$$\pi_{PPD} = \frac{120 * (1060 - 100)}{2} = 57600$$

Maximum willingness to pay for fee:

$$\text{fee} \leq \pi_{PPD} - \pi_{NPD} = 57600 - 28800 = 28800$$

6 Monopoly pricing 3

6.1 Problem

Demand functions of your two different customers S and P:

$$Q_S = 6 - 2P_S$$

$$Q_P = 14 - 2P_P$$

Constant marginal cost: $MC = 1$.
No fixed cost.

1. What would your profit be if you could only set one price for both customers together (uniform pricing)?
2. What would your profit be under direct segment discrimination using just a price per unit?
3. What would your profit be in you could set a two-part tariff for each customer, relying at the same time on direct segment discrimination?

6.2 Solution

1.

$$Q = Q_S + Q_P = 6 - 2P + 14 - 2P = 20 - 4P$$

Profit uniform pricing:

$$\pi = P(20 - 4P) - 1(20 - 4P) = -4P^2 + 24P - 20$$

First order condition:

$$-8P^* + 24 = 0$$

Profit-maximizing price and quantity:

$$P^* = 3,\ Q^* = 20 - 4 \cdot 3 = 8$$

Profit uniform pricing:

$$\pi = P^*Q^* - Q^* = 3 \cdot 8 - 8 = 16$$

2. Profits direct segment discrimination:

$$\pi_S = P_S(6 - 2P_S) - 1(6 - 2P_S) = -2P_S^2 + 8P_S - 6$$

$$\pi_P = P_P(14 - 2P_P) - 1(14 - 2P_P) = -2P_P^2 + 16P_P - 14$$

Profit-maximizing prices and quantities

$$-4P_S^* + 8 = 0 \Rightarrow P_S^* = 2 \Rightarrow Q_S^* = 2$$

$$-4P_P^* + 16 = 0 \Rightarrow P_P^* = 4 \Rightarrow Q_P^* = 6$$

Profit direct segment discrimination:

$$(2 \cdot 2 - 1 \cdot 2) + (4 \cdot 6 - 1 \cdot 6) = 20$$

3. Two-part tariff: Unit price = marginal cost, fixed fee = consumer surplus when price equals marginal cost. (Tip: Make a drawing.)

$$Q_S^* = 6 - 2 \cdot 1 = 4$$
$$Q_P^* = 14 - 2 \cdot 1 = 12$$

Fixed fees:

$$\text{fee}_S = \frac{4 \cdot (3-1)}{2} = 4$$
$$\text{fee}_P = \frac{12 \cdot (7-1)}{2} = 36$$

Profit two-part tariff + direct segment discrimination:

$$\pi = \text{fee}_S + 1 \cdot Q_S^* + \text{fee}_P + 1 \cdot Q_P^* - 1 \cdot (Q_P^* + Q_S^*) = 4 + 4 + 36 + 12 - (4 + 12) = 40$$

7 Bundling 1

7.1 Problem

Demand side:

Segment	# households	Educ. channel	Music channel	Bundle
Conservative	4000	\$20	\$2	\$22
Mainstream	6000	\$11	\$11	\$22

Constant marginal cost: MC

Maximize profits using

1. pure bundling (only sell bundle of two channels together)
2. separate prices (only sell each channel separately)
3. mixed bundling (combination of pure bundling and seperate priced)

Under what circumstances will you choose which pricing scheme, and what will your profits be?

7.2 Solution

1. The only price that you want to consider is 22, and hence the profits are

$$\pi^{PB} = 10000(22 - 2MC) = 220000 - 20000MC.$$

2a. The prices you want to consider are 2 and 11, and hence the profits are

$$\pi_M^{SP}(2) = 10000(2 - MC) = 20000 - 10000MC,$$

$$\pi_M^{QP}(11) = 6000(11 - MC) = 66000 - 6000MC.$$

As the revenues are larger, it is clear that the price for the Music channel is 11.

2b. The prices you want to consider are 11 and 20, and hence the profits are

$$\pi_E^{SP}(11) = 10000(11 - MC) = 110000 - 10000MC,$$

$$\pi_E^{SP}(20) = 4000(20 - MC) = 80000 - 4000MC.$$

You want to set price 11 if

$$\pi_E^{SP}(11) \geq \pi_E^{SP}(20),$$

$$\text{or } 110000 - 10000MC \geq 80000 - 4000MC$$

$$\text{or } 5 \geq MC.$$

Otherwise, you want to set price 20.

3. It is a safe bet that you want to sell only the Educational channel to the Conservatives, and both channels to the Mainstream households.
The highest price you can ask for the Educational channes is 20, and 22 for the bundle. Therefore, your profits would be

$$\pi^{MB} = 4000(20 - MC) + 6000(22 - 2MC) = 212000 - 16000MC.$$

You don't want to sell the Music channel separately, so you price it at 12.

Benefits for Conservatives:

Educ. channel	Music channel	Bundle
20-20 = 0	2-12 = -10	22-22=0

Benefits for Mainstream:

Educ. channel	Music channel	Bundle
11-20 = -9	11-12 = -1	**22-22=0**

answer. If $MC > 20$, you can't make profits. So you sell nothing.

If $MC > 11$, you want to choose separate prices and only sell to Conservatives.

Note that $\pi^{MB} > \pi^{SP}$ if $MC \leq 5$. Thus mixed bundling dominates separate prices.
If $MC \leq 2$, pure bundling gives higher profits than mixed bundling. If $MC > 2$, the opposite is true.

If $5 < MC \leq 11$, we need to compare mixed bundling profits to profits from separate prices. This yields

$$\pi^{MB} - \pi^{SP} = 212000 - 16000MC - (146000 - 10000MC).$$

This is positive if and only if $MC < 11$.

	$MC \leq 2$ PB	$2 < MC \leq 5$ MB	$5 < MC \leq 11$ MB	$11 < MC \leq 20$ SP	$MC > 20$ not selling
Price bundle	**22**	**22**	**22**	23	23
Price Music channel	12	12	12	12	12
Price Educ. channel	21	**20**	**20**	**20**	21

8 Bundling 2

8.1 Problem

Segment	# people	A	B	Bundle
Boys	7	5	8	13.5
Girls	3	7.5	6	13

Marginal cost: $MC = 2$

1. What would your profit be if you could only use pure bundling?
2. What would your profit be if you could only use separate prices?
3. What would your profit be if you could only use mixed bundling?

8.2 Solution

1.

$$\pi(13) = 10 \cdot (13 - 4) = 90$$

$$\pi(13.5) = 7 \cdot (13.5 - 4) = 66.5$$

Your profit would be 90.

2.

$$\pi_A(5) = 10 \cdot (5 - 2) = 30$$

$$\pi_A(7.5) = 3 \cdot (7.5 - 2) = 16.5$$

$$\pi_B(6) = 10 \cdot (6 - 2) = 40$$

$$\pi_B(8) = 7 \cdot (8 - 2) = 42$$

Your profit would be $30 + 42 = 72$.

3a. You want to sell bundle to boys and only A to girls.

Benefits:

	A	B	Bundle
Boys	$5 - P_A$ $5 - 7.5 = -2.5$	$8 - P_B$ $8 - 9 = -1$	$13.5 - P_{bundle}$ $13.5 - 13.5 = 0$
Girls	7.5-P_A $7.5 - 7.5 = 0$	6 - P_B $6 - 9 = -3$	13 - P_{bundle} $13 - 13.5 = -0.5$

$$\pi = 7 \cdot (13.5 - 4) + 3 \cdot (7.5 - 2) = 83$$

3b. You want to sell bundle to girls and only B to boys.

Benefits:

	A	B	Bundle
Boys	$5 - P_A$ $5 - 8 = -3$	$8 - P_B$ $8 - 7.5 = 0.5$	$13.5 - P_{bundle}$ $13.5 - 13 = 0.5$
Girls	7.5-P_A $7.5 - 8 = -0.5$	6 - P_B $6 - 7.5 = -1.5$	13 - P_{bundle} $13 - 13 = 0$

$$\pi = 3 \cdot (13 - 4) + 7 \cdot (7.5 - 2) = 32.5$$

3c. Your profit would be 83. It would be higher if you used pure bundling, so you use pure bundling by pricing A at 8 and B at 9.

9 Cournot competition

9.1 Problem

Inverse demand function: $P = 13 - Q$. (P = price, Q = quantity)
Constant marginal cost: $MC = 1$.
Two-stage game:

1. You have to pay $F > 0$ to enter the industry.

2. You face competitors and you all decide quantities (as in Cournot competition).

How large can F be if you expect to face N competitors in the second stage of the game?

9.2 Solution

Profit

$$\pi_i = PQ_i - Q_i = (P-1)Q_i = (12 - (Q_1 + \ldots + Q_{N+1}))Q_i$$

First order condition

$$\begin{aligned} & 12 - (Q_1 + \ldots + Q_{i-1} + Q_{i+1} + \ldots + Q_{N+1}) - 2Q_i = 0 \\ \Rightarrow \quad & Q_i = 6 - \frac{1}{2}(Q_1 + \ldots + Q_{i-1} + Q_{i+1} + \ldots + Q_{N+1}) \end{aligned}$$

Calculating quantity

$$\begin{aligned} & Q_1^* = \ldots = Q_i^* = \ldots = Q_{N+1}^* \\ \Rightarrow \quad & Q_i^* = 6 - \frac{1}{2}(NQ_i^*) \\ \Rightarrow \quad & \left(1 + \frac{N}{2}\right) Q_i^* = 6 \\ \Rightarrow \quad & \frac{N+2}{N} Q_i^* = 6 \\ \Rightarrow \quad & Q_i^* = \frac{12}{N+2} \end{aligned}$$

Calculating profit

$$\pi_i = \left(12 - (N+1)\frac{12}{N+2}\right)\frac{12}{N+2} = \frac{144}{(N+2)^2}$$

Answer

$$F \leq \frac{144}{(N+2)^2}$$