Ik bied geen garantie op de juistheid hiervan, maar het is misschien al een goeie start. Veel succes! W.

2023 januari

1 a) $T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2$

$q = (x_2 - x_1) - \ell = (x - x_1) - \ell$

$\Rightarrow -x_1 = q + \ell - x$

$\Rightarrow x_1 = x - \ell - q$

$V = \frac{1}{2} k q^2$

$L = T - V = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}^2 - \frac{1}{2} k q^2$

$= \frac{1}{2} m_1 (\dot{x} - \dot{q})^2 + \frac{1}{2} m_2 \dot{x}^2 - \frac{1}{2} k q^2$

↳ mag weg, want beweging ligt vast

$\frac{\partial L}{\partial \dot{q}} = -m_1(\dot{x} - \dot{q}) = m_1(\dot{q} - \dot{x})$

$\Rightarrow H = m_1(\dot{q} - \dot{x})\dot{q} - \frac{1}{2} m_1 \dot{x}^2 + m_1 \dot{x}\dot{q} - \frac{1}{2} m_1 \dot{q}^2 + \frac{1}{2} k q^2$

$= m_1 \dot{q}^2 - \cancel{m_1 \dot{x}\dot{q}} - \frac{1}{2} m_1 \dot{x}^2 + \cancel{m_1 \dot{x}\dot{q}} - \frac{1}{2} m_1 \dot{q}^2 + \frac{1}{2} k q^2$

$= \frac{1}{2} m_1 \dot{q}^2 - \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} k q^2$

$= \frac{p^2}{2m_1} + p\dot{x} + \frac{1}{2} k q^2$

$\dot{q} = \frac{\partial H}{\partial p} = \frac{p}{m_1} + \dot{x}$ en $\dot{p} = -\frac{\partial H}{\partial q} = -kq$

b) $\ddot{q} = \frac{\dot{p}}{m_1} + \ddot{x} = \frac{-kq}{m_1} + 6at = -\omega^2 q + 6at$

De gegeven oplossing voldoet aan deze vergelijking

c) $q(t) = \frac{6a}{\omega^2}\left(t - \frac{1}{\omega}\left(\omega t - \frac{1}{6}(\omega t)^3 + O(\omega t)^5\right)\right) = at^3$

② a) centrifugaalkracht + corioliskracht

b) snelheid < ontsnappingssnelheid
snelheid groot genoeg zodat pijltje niet op de grond valt

c) baan is ellips.

$$E = \frac{1}{2} m v_p^2 - G\frac{Mm}{R}$$

$$= \frac{1}{2} m (v^2 + R^2\Omega^2) - G\frac{Mm}{R}$$

$$E = -\frac{\cancel{k}}{2a} \cancel{\pm} \frac{GMm}{2a} \Rightarrow a = -\frac{GMm}{2E}$$

$$T^2 = \frac{(2\pi)^2}{GM}\left(\frac{GMm}{2|E|}\right)^3 = \left(\frac{2\pi}{\Omega}\right)^2$$

$\Rightarrow v^2 = \dots$

baan is cirkel

$$v_0 = \sqrt{\frac{GM}{R}}$$

$$\Rightarrow v = \sqrt{\frac{GM}{R} - R^2\Omega^2}$$

$\Rightarrow T = \dots$

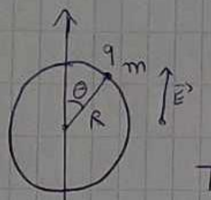
③

$$\frac{N \cdot r}{kg\,\frac{m^2}{r^2}} = \frac{kg\,\frac{m}{r^2}}{kg\,\frac{\cancel{mm}}{r^2}}$$

~~$x = \theta + r$~~

2022 Januari

$V(r) = b\left(\sqrt{r^2+a^2}+a\right)^2$

① 

$\vec{E} = E\hat{k}$

a) $T = \frac{1}{2}m\left(R^2\dot\theta^2 + R^2\sin^2\theta\,\dot\phi^2\right)$

$V = -qER\cos\theta$

$\Rightarrow L = \frac{1}{2}m\left(R^2\dot\theta^2 + R^2\sin^2\theta\,\dot\phi^2\right) + qER\cos\theta$

b) Energie is behouden want $\frac{\partial L}{\partial t} = 0$

$\frac{\partial L}{\partial \dot\phi} = mR^2\sin^2\theta\,\dot\phi$ is ook behouden, dit is de $\hat{k}$-component van $\vec{L}$

c) $\frac{d}{dt}\frac{\partial L}{\partial \dot\theta} = mR^2\ddot\theta = mR\sin\theta\cos\theta\,\dot\phi^2 - qER\sin\theta$

In "evenwicht" als $\ddot\theta = \dot\theta = 0$

$\Rightarrow m\sin\theta\cos\theta\dot\phi = qE\sin\theta$

$\Leftrightarrow \sin\theta = 0$ OF $\cos\theta = \frac{qE}{m\dot\phi^2}$

$\Leftrightarrow \theta = n\pi$

Deeltje zweeft op hoogte $R\cos\theta = \begin{cases} R & \text{als } \theta = n\pi \\ \frac{qER}{m\dot\phi^2} & \text{in ander geval} \end{cases}$

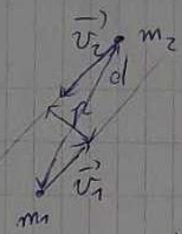
d) Stel $\theta = n\pi + \varphi$, φ is klein

$\Rightarrow L \approx \frac{1}{2}m\left(R^2\dot\varphi^2 + R^2\sin^2\varphi\,\dot\phi^2\right) - qER\frac{\varphi^2}{2}$

$\Rightarrow \frac{d}{dt}\frac{\partial L}{\partial \dot\varphi} = mR^2\ddot\varphi = mR^2\sin\varphi\cos\varphi\,\dot\phi^2 - qER\varphi$

$\Rightarrow \ddot\varphi = \left(\frac{1}{2}\sin(2\varphi)\,\dot\phi^2 - \frac{qE}{mR}\right)\varphi$

$\Rightarrow \omega^2 = \frac{1}{2}\sin(2\varphi)\,\dot\phi^2 - \frac{qE}{mR}$

② 

Niet gebonden als $E > 0$

Dus als $E_{kin} > E_{grav}$

$$\Rightarrow \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 > \frac{G m_1 m_2}{d}$$

③ a) $L = L_0 + \frac{df(q)}{dt}$

$$\frac{\partial L}{\partial \dot q^\alpha} = \frac{\partial L_0}{\partial \dot q^\alpha} + \frac{\partial}{\partial \dot q^\alpha}\frac{df(q)}{dt} \quad \text{en} \quad \frac{\partial L}{\partial q^\alpha} = \frac{\partial L_0}{\partial q^\alpha} + \frac{\partial}{\partial q^\alpha}\frac{df(q)}{dt}$$

$$\Rightarrow \frac{d}{dt}\frac{\partial L}{\partial \dot q^\alpha} = \frac{d}{dt}\frac{\partial L_0}{\partial \dot q^\alpha} + \frac{d}{dt}\frac{\partial}{\partial \dot q^\alpha}\frac{df(q)}{dt}$$

TB $\frac{d}{dt}\frac{\partial}{\partial \dot q^\alpha}\frac{df(q)}{dt} = \frac{\partial}{\partial q^\alpha}\frac{df(q)}{dt}$

$$\frac{d}{dt}\frac{\partial}{\partial \dot q^\alpha}\frac{df(q)}{dt} = \frac{d}{dt}\frac{\partial}{\partial \dot q^\alpha}\sum_\beta \frac{\partial f}{\partial q^\beta}\dot q^\beta$$

$$= \frac{d}{dt}\sum_\beta \frac{\partial f}{\partial q^\beta}\frac{\partial \dot q^\beta}{\partial \dot q^\alpha}$$

$$= \frac{d}{dt}\frac{\partial f}{\partial q^\alpha}$$

$$= \frac{\partial}{\partial q^\alpha}\frac{df(q)}{dt} \quad \blacksquare$$

b) $q^\alpha \to q^\alpha + \epsilon(t) n^\alpha(q)$

Er wordt enkel een tijdsafgeleide toegevoegd

Dus de beweging blijft gelijk dus behoud van energie

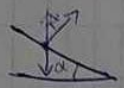
c) $$\frac{dQ}{dt} = \sum_\alpha \frac{\partial L}{\partial q^\alpha}\left(\frac{\partial q^\alpha_\epsilon}{\partial \epsilon}\right)_{\epsilon=0} + \frac{\partial L}{\partial \dot q^\alpha}\frac{d}{dt}\left(\frac{\partial q^\alpha_\epsilon}{\partial \epsilon}\right)_{\epsilon=0} \stackrel{?}{=} 0 \qquad q^\alpha_n = q^\alpha + \epsilon(t) n^\alpha(q)$$

$$= \sum_\alpha \frac{\partial L}{\partial q^\alpha}\cdot 0 + \frac{\partial L}{\partial \dot q^\alpha}\cdot 0$$

$$= 0 \quad ??$$

2021 januari

① a) De versnelling komt door normaalkracht



$F_n = \frac{mg}{\cos\alpha}$ en $F_{\to} = F_n \sin\alpha$

$\Rightarrow F_{\to} = mg\tan\alpha$

b) Lagrangiaan van een deeltje in een magneetveld:

$$L = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + e\vec{v}\cdot\vec{A}$$

$$= \frac{1}{2} m\dot{\vec{r}}^2 + e\dot{\vec{r}}\cdot\vec{A}$$

Omzetten nr Hamiltoniaan

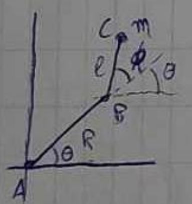
$$\frac{\partial L}{\partial \dot{\vec{r}}} = m\dot{\vec{r}} + e\vec{A} = p_r \Rightarrow \dot{\vec{r}} = \frac{p_r - e\vec{A}}{m}$$

$$\Rightarrow H = m\dot{\vec{r}}^2 + e\vec{A}\cdot\dot{\vec{r}} - \frac{1}{2}m\dot{\vec{r}}^2 - e\dot{\vec{r}}\cdot\vec{A}$$

$$= \frac{1}{2}m\dot{\vec{r}}^2$$

$$= \frac{(p_r - e\vec{A})^2}{2m}$$ is niet even in p

② a)



$$\vec{r} = (R\cos\theta + l\cos(\theta+\phi),\ R\sin\theta + l\sin(\theta+\phi))$$

$$\dot{\vec{r}} = (-R\sin\theta\dot\theta - l\sin(\theta+\phi)(\dot\theta+\dot\phi),\ R\cos\theta\dot\theta + l\cos(\theta+\phi)(\dot\theta+\dot\phi))$$

$$L = \frac{1}{2}m\dot{\vec{r}}^2 = \frac{m}{2}\Big(R^2\sin^2\theta\,\dot\theta^2 + Rl\sin\theta\sin(\theta+\phi)(\dot\theta+\dot\phi)\dot\theta + l^2\sin^2(\theta+\phi)(\dot\theta+\dot\phi)^2 + R^2\cos^2\theta\,\dot\theta^2 + Rl\cos\theta\cos(\theta+\phi)(\dot\theta+\dot\phi)\dot\theta + l^2\cos^2(\theta+\phi)(\dot\theta-\dot\phi)^2\Big)$$

$$\Rightarrow L = \frac{m}{2}\Big(R^2\dot\theta^2 + Rl(\sin\theta\sin(\theta+\phi) + \cos\theta\cos(\theta+\phi))(\dot\theta+\dot\phi)\dot\theta + l^2(\dot\theta+\dot\phi)^2\Big)$$

$$= \frac{m}{2}\Big(R^2\dot\theta^2 + Rl\cos\phi(\dot\theta+\dot\phi)\dot\theta + l^2(\dot\theta+\dot\phi)^2\Big)$$

$$L = \frac{m}{2}\left(R^2\dot\theta^2 + Rl\cos\phi\,(\dot\theta+\dot\phi)\dot\theta + l^2(\dot\theta+\dot\phi)^2\right)$$

$$\frac{\partial L}{\partial\dot\phi} = \frac{m}{2}Rl\cos\phi\,\dot\theta + ml^2(\dot\theta+\dot\phi)$$

$$\frac{d}{dt}\frac{\partial L}{\partial\dot\phi} = -\frac{m}{2}Rl\sin\phi\,\dot\phi\,\dot\theta + \cancel{\frac{m}{2}Rl\cos\phi\,\ddot\theta} + ml^2(\ddot\theta+\ddot\phi), \quad \ddot\theta = 0$$

$$\frac{\partial L}{\partial\phi} = -\frac{m}{2}Rl\sin\phi\,(\dot\theta+\dot\phi)\dot\theta$$

$$\frac{\partial L}{\partial\dot\theta} = mR^2\dot\theta + \frac{m}{2}Rl\cos\phi(2\dot\theta+\dot\phi) + ml^2(\dot\theta+\dot\phi) \text{ is behouden}$$

b) BC in verlengde AB als $\phi(0)=0$ en $\dot\phi=0$

$r^2 = R^2 + l^2$ altijd als $\phi = 0$

Geen idee hoe ge dit bewijst tho

c) Stel ϕ is klein en $\dot\phi$ klein

$$L \approx \frac{m}{2}\left(R^2\dot\theta^2 + Rl\left(1-\frac{\phi^2}{2}\right)(\dot\theta+\dot\phi)\dot\theta + l^2(\dot\theta+\dot\phi)^2\right)$$

$$\approx \frac{m}{2}\left(R^2\dot\theta^2 - Rl\frac{\phi^2}{2}\dot\theta^2 + l^2(\dot\theta+\dot\phi)^2\right)$$

$$\Rightarrow \frac{\partial L}{\partial\phi} = -\frac{m}{2}Rl\phi\dot\theta^2 \quad \text{en} \quad \frac{d}{dt}\frac{\partial L}{\partial\dot\phi} = ml^2\ddot\phi \quad \text{want } \ddot\theta = 0$$

$$\Rightarrow \ddot\phi = \frac{-R\dot\theta^2}{2l}\phi \rightarrow \text{exponentiele functie}$$

Een kleine verandering verstoort heel het evenwicht.

③ cirkels: $v = \omega R$ $F_{wrijving} = \mu F_N = \mu m g$

$a = \frac{v^2}{R}$

=> Maximale v als $F_{centrifugaal} = F_{wrijving}$

=> $\cancel{m \frac{v^2}{R} = \mu m g}$

=> $\cancel{v_{max} = \sqrt{\mu R g}}$ ~~tov draaitafel~~

=> $\cancel{\omega_{max} = \sqrt{\frac{\mu g}{R}}}$

=> $\cancel{m} \frac{(\omega R + v)^2}{R} = \mu \cancel{m} g$

=> $v = \sqrt{\mu R g} - \omega R$ tov draaitafel als hij met draairichting loopt

$v = \sqrt{\mu R g} + \omega R$ als hij tegen tafel inloopt

2021, 19 augustus

① $H(f_1(q_1,p_1), f_2(q_2,p_2), \dots, f_n(q_n,p_n))$

$$\frac{\partial H}{\partial t} = 0$$

$$H = \sum_{a=1}^{n} f_a(q_\alpha, p_\alpha) \Rightarrow \quad \dot{p}_\alpha = -\frac{\partial f_\alpha(q_\alpha, p_\alpha)}{\partial q_\alpha}$$

$$\dot{q}_\alpha = \frac{\partial f_\alpha(q_\alpha, p_\alpha)}{\partial p_\alpha}$$

$$\frac{d f_\alpha(q_\alpha, p_\alpha)}{dt} = \frac{\partial f_\alpha}{\partial q_\alpha}\dot{q}_\alpha + \frac{\partial f_\alpha}{\partial p_\alpha}\dot{p}_\alpha$$

$$= \frac{\partial f_\alpha}{\partial q_\alpha}\frac{\partial f_\alpha}{\partial p_\alpha} - \frac{\partial f_\alpha}{\partial p_\alpha}\frac{\partial f_\alpha}{\partial q_\alpha}$$

$$= 0 \quad \boxtimes$$

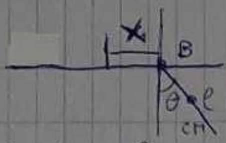
② Unbound als $E > 0$

$$K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 > G\frac{m_1 m_2}{d}$$

$$G' = \frac{m_2}{m_1 + m_2} G$$, V is relatief

$$\frac{1}{2}\cancel{m_1} V^2 > \frac{\cancel{m_1} m_2}{d} G'$$

$$\Rightarrow V^2 > 2\frac{m_1 m_2}{d(m_1+m_2)} G$$

③ 

$x = A\sin(\omega t)$, $\dot{x} = A\omega\cos(\omega t)$

$\ddot{x} = -\omega x$

a) $\vec{r} = (x + \frac{\ell}{2}\sin\theta , \frac{\ell}{2}\cos\theta)$

$\dot{\vec{r}} = (\dot{x} + \frac{\ell}{2}\cos\theta\dot{\theta} , -\frac{\ell}{2}\sin\theta\dot{\theta})$

$\Rightarrow T = \frac{1}{2}m\dot{\vec{r}}^2 = \frac{m}{2}(\dot{x}^2 + \ell\cos\theta\,\dot{x}\dot{\theta} + \frac{\ell^2}{4}\dot{\theta}^2)$

$V = -\frac{mg\ell}{2}\cos\theta$

$\Rightarrow L = T - V = \frac{m}{2}(\dot{x}^2 + \ell\cos\theta\,\dot{x}\dot{\theta} + \frac{\ell^2}{4}\dot{\theta}^2) + mg\frac{\ell}{2}\cos\theta$

of

$L = \frac{m}{2}(A^2\omega^2\cos^2(\omega t) + \ell\cos\theta A\omega\cos(\omega t)\dot{\theta} + \frac{\ell^2}{4}\dot{\theta}^2) + \frac{\ell}{2}\cos\theta$

$\frac{\partial L}{\partial\dot{\theta}} = \frac{m}{2}\ell\cos\theta A\omega\cos(\omega t) + \frac{\ell^2}{2}\dot{\theta} = \frac{m}{2}\ell\cos\theta\,\dot{x} + \frac{\ell^2}{2}\dot{\theta}$

$\frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}} = -\frac{m}{2}\ell\sin\theta\dot{\theta}\dot{x} + \frac{m}{2}\ell\cos\theta\,\ddot{x} + \frac{\ell^2}{2}\ddot{\theta}$

$\frac{\partial L}{\partial\theta} = -\frac{m}{2}\ell\sin\theta\,\dot{x}\dot{\theta} - \frac{\ell}{2}\sin\theta$, vaag gedoe

b) Stel θ klein

$L \approx \frac{m}{2}(\dot{x}^2 + \ell(1-\frac{\theta^2}{2})\dot{x}\dot{\theta} + \frac{\ell^2}{4}\dot{\theta}^2) - \frac{\ell^2}{4}\theta^2$

$\frac{\partial L}{\partial\dot{\theta}} = \frac{m}{2}\ell(1-\frac{\theta^2}{2})\dot{x} + \frac{\ell^2}{2}\dot{\theta}$ en $\frac{\partial L}{\partial\theta} = \frac{m}{2}\ell(-\theta)\dot{x}\dot{\theta} - \frac{\ell^2}{2}\theta$

$\Rightarrow \frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}} = \frac{m}{2}\ell(-\theta\dot{\theta})\dot{x} + \frac{m}{2}\ell(1-\frac{\theta^2}{2})\ddot{x} + \frac{\ell^2}{2}\ddot{\theta}$

$\Rightarrow \frac{m}{2}\ell(1-\frac{\theta^2}{2})\ddot{x} + \frac{\ell^2}{2}\ddot{\theta} = -\frac{\ell^2}{2}\theta$

$\Rightarrow \ddot{\theta} = -\theta - \frac{m}{\ell}(1-\frac{\theta^2}{2})\ddot{x}$

2021, 17 Augustus

① $H = \sum_{i=1}^{3} \alpha_i p_i^2 + V = \sum_{i=1}^{3} \frac{\partial L}{\partial \dot{q}^i} \dot{q}^i - T + V$

$\Rightarrow T = \sum_{i=1}^{3} \frac{\partial L}{\partial \dot{q}^i} \dot{q}^i - \alpha_i p_i^2$

met $\frac{\partial L}{\partial \dot{q}^i} = p_i$, $\dot{q}^i = \frac{\partial H}{\partial p_i} = 2\alpha_i p_i$

$\Rightarrow T = \sum_{i=1}^{3} 2\alpha_i p_i^2 - \alpha_i p_i^2$

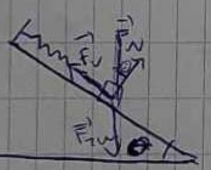
$= \sum_{i=1}^{3} \alpha_i p_i^2$

$\Rightarrow L = \sum_{i=1}^{3} \alpha_i p_i^2 - V(q_1^2 + q_2^2 + q_3^2) = \alpha(p_1^2 + p_2^2 + p_3^2) - V(q_1^2 + q_2^2 + q_3^2)$

$\frac{\partial L}{\partial t} = 0$ dus behoud v. energie

Alle $\frac{\partial L}{\partial p_i}$ zijn behouden want $\frac{\partial L}{\partial p_i} = 0$

② Zie examen vorige jaren

③ a) 

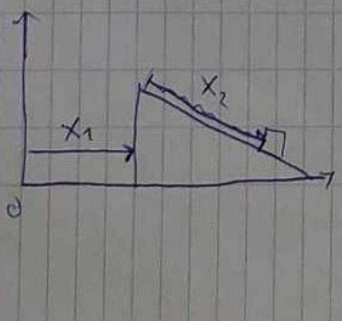
$F_{zw} = mg$

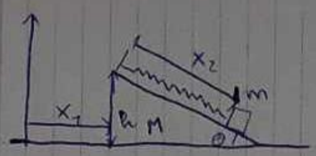
$F_N \cos\theta = mg$ $\left(\Rightarrow F_N = \frac{mg}{\cos\theta}\right)$

$F_{zw} \sin\theta = F_v \Rightarrow F_v = k(x_2 - d) = mg \sin\theta$

$\Rightarrow F_v = k(x - d) \Rightarrow x_{ev} = \frac{mg}{k} \sin\theta + d$

b) 2 vrijheidsgraden





$$V = -mg\,x_2 \sin\theta + \frac{1}{2}k(x_2-d)^2$$

$$K_M = \frac{1}{2}M\dot{x}_1^2$$

$$K_m = \frac{1}{2}m(\dot{x}_1 + \dot{x}_2(\cos\theta))^2 + \frac{1}{2}m(\dot{x}_2\sin\theta)^2$$

$$= \frac{m}{2}(\dot{x}_1^2 + 2\dot{x}_1\dot{x}_2\cos\theta + \dot{x}_2^2)$$

$$\Rightarrow L = \frac{1}{2}M\dot{x}_1^2 + \frac{m}{2}(\dot{x}_1^2 + 2\dot{x}_1\dot{x}_2\cos\theta + \dot{x}_2^2) + mg\,x_2\sin\theta - \frac{1}{2}k(x_2-d)^2$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_2} = m\ddot{x}_1\cos\theta + m\ddot{x}_2 = mg\sin\theta - k(x_2-d)$$

$$\Rightarrow \quad \ddot{x}_1\cos\theta + \ddot{x}_2 = g\sin\theta - \frac{k}{m}(x_2-d)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_1} = M\ddot{x}_1 + m\ddot{x}_1 + m\ddot{x}_2\cos\theta = 0$$ → Alle krachten heffen mooi op

c) Gaat niet via T en V, want V bestaat precies niet ?

Stel $x_2 = x_{ev} + s_2$ met $s_{1,2}$ klein , $x_{ev} = \frac{mg}{k}\sin\theta + d$

$x_1 = s_1$

$$\Rightarrow L \approx \frac{1}{2}M\dot{s}_1^2 + \frac{m}{2}(\dot{s}_1^2 + 2\dot{s}_1\dot{s}_2\cos\theta + \dot{s}_2^2) + mg\,s_2\sin\theta - \frac{1}{2}k(s_2^2 + 2\frac{mg}{k}\sin\theta\, s_2)$$

$$\Rightarrow L \approx \frac{1}{2}M\dot{s}_1^2 + \frac{m}{2}(\dot{s}_1^2 + 2\dot{s}_1\dot{s}_2\cos\theta + \dot{s}_2^2) - \frac{1}{2}k s_2^2$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{s}_1} = (M+m)\ddot{s}_1 + m\ddot{s}_2\cos\theta = 0 \Rightarrow \ddot{s}_1 = \frac{-m\cos\theta}{M+m}\ddot{s}_2$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{s}_2} = \cancel{m}\ddot{s}_2 + \cancel{m}\ddot{s}_1\cos\theta = -\frac{k}{m}s_2$$

$$\Rightarrow \ddot{s}_2\left(1 - \frac{m\cos^2\theta}{M+m}\right) = -\frac{k}{m}s$$

$$\Rightarrow \omega^2 = \frac{k}{m\left(\frac{m\cos^2\theta}{M+m} - 1\right)}$$

2020, 17 januari

(1) Botsingsdoorsnede $A = \int_S D(\theta,\phi) \sin\theta \, d\theta d\phi$ in m^2

$$A = 2\pi \int_{-\pi/4}^{\pi/4} \frac{k^2 q_1^2 q_2^2}{16 E^2} \frac{1}{\sin^4 \theta/2} \sin\theta \, d\theta$$

(2) zie vorig examen

(3) $\vec{F} = -\frac{\alpha}{r^3}\hat{r} = -\alpha u^3 \hat{r}$, $\frac{1}{2} m \dot{r}^2 = \frac{1}{2} m \left(\frac{dr}{d\theta}\right)^2 \left(\frac{d\theta}{dt}\right)^2$ en $L = mr^2\dot{\theta}$

$\Rightarrow V(r) = -\frac{\alpha}{2r^2}$, $E = \frac{L^2}{2mr^4}\left(\frac{dr}{d\theta}\right)^2 + \frac{L^2/m - \alpha}{2r^2}$

Padvergelijking: $\frac{d^2u}{d\theta^2} + u = + \frac{m\alpha u^3}{L^2 u^2} = \frac{m\alpha}{L^2} u$

$\Rightarrow \frac{d^2u}{d\theta^2} = \left(\frac{m\alpha}{L^2} - 1\right) u$

Stel $\frac{m\alpha}{L^2} - 1 = 0$, dus $\alpha = \frac{L^2}{m}$

$\Rightarrow u = A\theta + B \Rightarrow r = \frac{1}{A\theta + B}$, is cirkel als $A = 0$
spiraal als $A \neq 0$

Stel $\frac{m\alpha}{L^2} - 1 < 0$, dus $\alpha < \frac{L^2}{m}$

$\Rightarrow \frac{d^2u}{d\theta^2} = -\omega_1^2 u \Rightarrow u = A\sin\omega\theta + B\cos\omega\theta$ met $\omega_1^2 = 1 - \frac{m\alpha}{L^2}$

$\Rightarrow r = \frac{1}{A\sin\omega\theta + B\cos\omega\theta}$

Stel $\frac{m\alpha}{L^2} - 1 > 0$, dus $\alpha > \frac{L^2}{m}$

$\Rightarrow \frac{d^2u}{d\theta^2} = \omega_2^2 u \Rightarrow u = A\cosh\omega_2\theta + B\sinh\omega_2\theta$ met $\omega_2^2 = \frac{m\alpha}{L^2} - 1$

$\Rightarrow r = \frac{1}{A\cosh\omega_2\theta + B\sinh\omega_2\theta}$

(4) $\left.\begin{array}{l}\text{verschuiving } x \\ \text{verschuiving } y\end{array}\right\}$ en combinaties $\rightarrow \begin{cases} x \rightarrow x + \lambda_1 \\ y \rightarrow y + \lambda_2 \end{cases}$

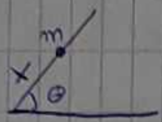
$\cancel{x \rightarrow \sqrt{2}\, y}$ Tijdreversibel

$\cancel{y \rightarrow \frac{x}{\sqrt{2}}}$

2019, 25 januari

① Nee

②



$\vec{r} = (x\cos\theta,\ x\sin\theta)$

$\dot{\vec{r}} = (\dot{x}\cos\theta - x\sin\theta\,\dot{\theta},\ \dot{x}\sin\theta + x\cos\theta\,\dot{\theta})$

$$T = \frac{1}{2} m\dot{\vec{r}}^2 = \frac{m}{2}\Big(\dot{x}^2\cos^2\theta - 2x\dot{x}\sin\theta\cos\theta\,\dot{\theta} + x^2\sin^2\theta\,\dot{\theta}^2 + \dot{x}^2\sin^2\theta + 2x\dot{x}\sin\theta\cos\theta\,\dot{\theta} + x^2\cos^2\theta\,\dot{\theta}^2\Big)$$

$$\Rightarrow T = \frac{m}{2}(\dot{x}^2 + x^2\dot{\theta}^2)$$

$$\Rightarrow L = \frac{m}{2}(\dot{x}^2 + x^2\dot{\theta}^2)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = m\ddot{x} = m x\dot{\theta}^2 \Rightarrow \ddot{x} = m\omega^2 x$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = m x^2\ddot{\theta} = 0$$ $\rightarrow$ behoud v. impulsmoment

$$\Rightarrow m x^2\dot{\theta} = L_i = m d^2\omega$$

$$\Rightarrow x = A e^{\sqrt{m}\,\omega t} + B e^{-\sqrt{m}\,\omega t}$$ en $x(0) = d$

$$\Rightarrow x = d e^{\sqrt{m}\,\omega t}$$

③ zie vorig examen

Die botsingsdoorsneden zijn kapot moeilijk

(4) 1. Ja
2. Neen
3. Neen, niet inerte stelsels
4. Ja
5. Repellor
6. Neen