VRAAG 1 (2.5 PTS)

Evalueer $I = \int \int_D \frac{dxdy}{(1+x^2+y^2)^2}$ met D één lus van de lemniscaatkromme (Adams p. 486).

Oplossing:

$$I = \int \int_D \frac{dxdy}{(1+x^2+y^2)^2} = \int_{-\pi/4}^{\pi/4} \int_0^{\sqrt{\cos 2\theta}} \frac{rdrd\theta}{(1+r^2)^2}$$

$$\stackrel{t=1+r^2}{=} \int_{-\pi/4}^{\pi/4} \int_1^{1+\cos 2\theta} \frac{dt}{2t^2} = \int_0^{\pi/4} \left(1 - \frac{1}{1+\cos 2\theta}\right) d\theta$$

$$= \int_0^{\pi/4} 1 d\theta - \int_0^{\pi/4} \frac{1}{2\cos^2\theta} d\theta$$

$$= \frac{\pi}{4} - \frac{1}{2} \int_0^{\pi/4} \sec^2\theta d\theta = \frac{\pi}{4} - \frac{1}{2}$$

VRAAG 2 (2.5 PTS)

Evalueer $I = \int_0^3 \left\{ \int_0^4 \left\{ \int_{x=y/2}^{x=(y/2)+1} \left(\frac{2x-y}{2} + \frac{z}{3} \right) dx \right\} dy \right\} dz$, in (a) rechtstreeks (1 punt) en in (b) met de onderstaande transformatie (1.5 punt):

$$u = \frac{2x - y}{2}$$
$$v = y/2$$
$$w = z/3$$

(a) Oplossing:

$$I = \int_0^3 \left\{ \int_0^4 \left\{ \int_{x=y/2}^{x=(y/2)+1} \left(\frac{2x-y}{2} + \frac{z}{3} \right) dx \right\} dy \right\} dz$$

$$= \int_0^3 \int_0^4 \left(\frac{x^2}{2} - \frac{xy}{2} + \frac{xz}{3} \right) \Big|_{x=y/2}^{x=(y/2)+1} dy dz$$

$$= \int_0^3 \int_0^4 \left(\frac{1}{2} + \frac{z}{3} \right) dy dz$$

$$= 4 \int_0^3 \left(\frac{1}{2} + \frac{z}{3} \right) dz = 12$$

(b) Oplossing:

$$x = u + y/2 = u + v$$
$$y = 2v$$
$$z = 3w$$
$$\Rightarrow dV = \left| \frac{\partial(x,y,z)}{\partial(u,v,w)} \right| = 6\, du\, dv\, dw$$

$$I = \int_0^3 \left\{ \int_0^4 \left\{ \int_{x=y/2}^{x=(y/2)+1} \left(\frac{2x-y}{2} + \frac{z}{3} \right) dx \right\} dy \right\} dz$$

$$= 6 \int_0^1 \int_0^2 \int_0^1 (u+w)\, du\, dv\, dw = 6 \int_0^1 \int_0^2 \left(\frac{u^2}{2} + wu \right) \Big|_0^1 dv\, dw = 6 \int_0^1 \int_0^2 \left(\frac{1}{2} + w \right) dv\, dw$$

$$= 12 \int_0^1 \left(\frac{1}{2} + w \right) dw = 12$$

VERBETERING

Verbetersleutel:

- "Domme" fouten: -0.5 punt . Voorbeeld: tekenfout, vergeten coëfficiënt, etc...
- "Zware" fouten: -1.0 tot -1.5 punt: fouten in Jacobiaan, fouten in bepaling van integratiegrenzen, foute trigonometrische transformaties, etc...
- Onvolledige opgave: -X afhankelijk van volledigheid