

EXAMEN WISK

(2) $f(x) = \ln(\sin x + \cos x)$

(a) dom = $\left] -\frac{\pi}{4} + 2k\pi, \frac{3\pi}{4} + 2k\pi \right[\quad k \in \mathbb{Z}$

$\ln x \rightarrow]0, +\infty[$ dus $\sin x + \cos x > 0$

$\Leftrightarrow \sin x > -\cos x$

$\Leftrightarrow \tan x > -1$

$\Leftrightarrow -\frac{\pi}{4} + 2k\pi < x < \frac{3\pi}{4} + 2k\pi$

(b) $f \rightarrow$ extremum waar $\sin x + \cos x \rightarrow$ extr

$\Rightarrow \sin x = \cos x$

$\Leftrightarrow \tan x = 1$

$\Rightarrow x = \frac{\pi}{4} + 2k\pi$ of $x = \frac{5\pi}{4} + 2k\pi$

extrema

$\notin \text{dom}$

$\Downarrow$
 $(\sin x + \cos x)'' \Rightarrow -\sin\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right)$
 $= -\sqrt{2} < 0$

$\Downarrow$
max

(c) bereik $\lim_{x \rightarrow \pi/4} \sin x + \cos x = 0$

$\nearrow f\left(\frac{\pi}{4}\right) = \ln(\sqrt{2}) = \frac{1}{2} \ln 2$
ber $f = \left] -\infty, \frac{1}{2} \ln 2 \right]$

⑤

$$f(x, y) = x e^y$$

$$\text{grad } f = (e^y, a e^y)$$

(b) stat. pnt zoeken waar we zoeken nl lok min/max

(c) $\theta \in [0, 2\pi[$; $D_\theta f = 0$ in $(1, -1)$?

$$\begin{aligned} \rightarrow D_\theta &= \cos \theta \frac{\partial f}{\partial x}(1, -1) + \sin \theta \frac{\partial f}{\partial y}(1, -1) \\ &= \cos \theta e^{-1} + \sin \theta e^{-1} \end{aligned}$$

$$\begin{aligned} \hookrightarrow &= 0 \Leftrightarrow \cos \theta = -\sin \theta \\ &\theta = \frac{3\pi}{4} + k\pi \end{aligned}$$

$$\text{Door beperking } \theta \rightarrow \theta = \frac{3\pi}{4} \text{ en } \frac{7\pi}{4}$$

(d) $y = -x$ (beperkt op rechte), stat pnt $(1, -1)$

$\hookrightarrow$ met methode Lagrange of substit

$y = -x$ subs. in f

$$f(x) = x e^{-x}$$

$$f'(x) = e^{-x} - x e^{-x} = e^{-x} (1 - x)$$

We vinden stat pnt door $f'(x) = 0$ te berekenen.

$\rightarrow$ afg zal 0 w vr $(1 - x) = 0$ of $x = 1$

$\rightarrow$ stat pnt $(1, -1)$

① $z \in \mathbb{C}$: vind alle opl v d vgl.

$$e^{\pi z^2} = 1$$

schrijf opl op in vorm $z = x + iy$

$$x'(t) = (x(t) \cdot t)^{\frac{2}{3}}$$

④

(a) $x_0 \in \mathbb{R}$, beginvw $x(0) = x_0$; geef ope $\forall x_0$ waarvoor er een ope bestaat.

DV ope'en

$$x(t) = \left(\frac{t^{\frac{5}{3}}}{5} + \frac{C}{3} \right)^3 \quad K \swarrow \quad 3x^{\frac{1}{3}} = 3 \frac{t^{\frac{5}{3}}}{5} + C$$

$$\text{BV: } x_0 = (0 + K)^3 \Rightarrow K = \sqrt[3]{x_0} \Rightarrow x(t) = \left(\frac{t^{\frac{5}{3}}}{5} + x_0^{\frac{1}{3}} \right)^3$$

$\rightarrow$ gn ope $\forall x_0 \in \mathbb{R}$

(b) unieke ope $\forall$ ~~RL~~ DV die $x(0) = 0$ voldoet?

$\rightarrow$ (a) : $x \neq 0$ als BVW $x(0) = 0$, dan de ju $x(t) = 0$ ook ope DV.

$\rightarrow$ (a) : $x(t) = \left(\frac{1}{5} t^{\frac{5}{3}} \right)^3$ is ope.

$\Rightarrow$ 2 ope, dus gn unieke ope

(c) $f^u \nearrow \Leftrightarrow x'(t) \geq 0$

$$\downarrow x'(t) = \left(\sqrt[3]{x(t) \cdot t} \right)^2$$

$\rightarrow$ RL is kwadraat $\Rightarrow x'(t) \geq 0 \forall t$

~~RL~~

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Wisk - herhalingsles

 $|\vec{a}|$ $||\vec{a}||$ ④ Inductie[VB] $\forall n \geq 1$

$$1^3 + \dots + n^3 = \frac{1}{4} n^2 (n+1)^2$$

$$* n=1: LL = 1; RL = \frac{1}{4} 2^2 = 1 \quad \checkmark$$

$$* n \rightarrow n+1:$$

$$\underbrace{1^3 + \dots + n^3}_{\frac{1}{4} n^2 (n+1)^2} + (n+1)^3 =$$

$$= (n+1)^2 \left(\underbrace{\frac{1}{4} n^2 + n + 1}_{\frac{1}{4} (n^2 + 4n + 4)} \right)$$

$$= \frac{1}{4} (n+1)^2 (n+2)^2$$

④ 1^o orde DV[VB] oef. 10.14 c

$$x'(t) + \frac{2t}{1+t^2} x = t^2$$

~~$$x'(t) + \frac{2t}{1+t^2} x = t^2$$~~

① Homogene DV: $\frac{dx}{dt} + \frac{2t}{1+t^2} x = 0$

$x(t) = 0$ is de op.

$$\frac{dx}{x} = - \frac{2t}{1+t^2} dt$$

$$\Rightarrow \underbrace{\int \frac{dx}{x}}_{= \ln|x|} = - \int \frac{2t}{1+t^2} dt = -\ln(1+t^2) + C = \ln\left(\frac{1}{1+t^2}\right) + C$$

$$\Leftrightarrow |x| = \frac{1}{1+t^2} \cdot e^C$$

$$\Rightarrow x(t) = \pm \frac{1}{1+t^2} \cdot \underbrace{e^C}_{= C}$$

~~allgemeine~~ op: $x(t) = C \frac{1}{1+t^2}$

② Inhomogene DV.

problem $x(t) = y(t) \cdot \left(\frac{1}{1+t^2}\right) = (1+t^2)^{-1}$

Vu in

$$\left(y' \cdot \frac{1}{1+t^2} + y \cdot \underbrace{\left(-\frac{1}{1+t^2} \cdot 2t\right)}_{\text{fallen weg.}} \right) + \underbrace{\frac{2t}{1+t^2} y \frac{1}{1+t^2}}_{= t^2} = t^2$$

$$\Leftrightarrow y' \frac{1}{1+t^2} = t^2$$

$$\Leftrightarrow y' = t^2 (1+t^2) = t^2 + t^4$$

$$\Leftrightarrow y(t) = \frac{t^3}{3} + \frac{t^5}{5} + C$$

~~allg~~ op. VR DV (opgave):

$$x(t) = \left(\frac{t^3}{3} + \frac{t^5}{5} + C \right) \frac{1}{1+t^2}$$

② Lineaire 2^e orde DV

[VB] Voor $c = -2$, $c = 1$ en $c = 2$

$$x''(t) + 2x'(t) + cx(t) = \cos(t)$$

① Homogene: $x'' + 2x' + cx = 0$

$$\lambda^2 + 2\lambda + c = 0$$

$$\Leftrightarrow \lambda = \frac{-2 \pm \sqrt{4 - 4c}}{2} = -1 \pm \sqrt{1 - c}$$

* $c = -2$ dan $\lambda = -1 \pm \sqrt{3}$

dan zijn $e^{(-1 \pm \sqrt{3})t}$ en lineair
onafh. op. 'en.

- [H1] Ijntallen
 - $\mathbb{C}$
 - bewijs v. volledige inductie
- [H2] Functies
- [H3] Transcendente functies
 - exp. functie in $\mathbb{C}$ -vlak (e^z)
- [H4] Afgeleiden
- [H5] Toepassingen v. afgeleiden
 - berekening lim.
- [H6] Integralen
 - oneigenlijke integralen
 - Riemann-integraal
 - Trapeziumregel
- [H7] Toepassingen v. integralen
- [H8] Integratietechnieken
- [H9] Functies v. 2 of meer veranderlijken
 - raaklijn /- vlak + richtingsafgeleide
 - stelling v. Schwarz & Young

[H10] Eerste orde DV

→ scheidbare veranderlijken

→ lin. DV 1^e orde

[H11] Lineaire DV v. 2^e orde

→ de coeff.

→ niet-homogene DV

[H12] Extrema v. functies v. meerdere veranderl.

→ zadelpunten

→ optimalisatie voorwaarden

[H13] Vectoren

⑤ K is niveaukromme $f(x, y) = 0$ van

$$f(x, y) = (x^2 + y^2)^2 - 6x^2y$$

$$r = \sqrt{x^2 + y^2}$$

a) K in polcoörd. : $K : r = 3 \sin(2\theta) \cos(\theta)$ met $\theta \in [0, \pi]$

$$K: f(x, y) = 0 \Leftrightarrow (x^2 + y^2)^2 = 6x^2y$$

$$\frac{dx}{dt} = 0 : \left(\frac{\partial z}{\partial x} \right) \frac{dx}{dt} + \left(\frac{\partial z}{\partial y} \right) \frac{dy}{dt} = 0$$

als $z = f(x, y)$, dan :

$$\frac{dz}{d\theta} = \frac{\partial z}{\partial x} \frac{dx}{d\theta} + \frac{\partial z}{\partial y} \frac{dy}{d\theta}$$

" -y " x

$$\Rightarrow \frac{dz}{d\theta} = -y \frac{\partial z}{\partial x} + x \frac{\partial z}{\partial y}$$

$$x = r \cos \theta \text{ en } y = r \sin \theta$$

Bewijs v. volledige inductie: $P(n)$ met $n \in \mathbb{N}_0$

① Basisstap voor $P(1)$

② Inductiestap : * Inductiehypothese :
veronderstel $P(k)$ is waar
* Te bewijzen : $P(k+1)$ is waar
* Bewijs

③ Conclusie

H2

$$f(x) = ax + b \quad \text{of} \quad f(x) = a(x - x_0)$$

$$\text{met } a = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{en } x_0 = \text{nulpunt} = -\frac{b}{a}$$

$b =$ snijpunt y -as

$$f(x) = ax^2 + bx + c \quad \text{met } a \neq 0$$

$$\text{of } f(x) = a(x - x_1)(x - x_2)$$

$$\text{met } x_{1,2} = \text{nulpunten} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\leadsto$ nulpunt $x = 1 \Rightarrow$ som coëff. = 0

nulpunt $x = -1 \Rightarrow$ som coëff. oneven macht
= som coëff. even macht

H3

podwórk:

$$x = r \cos \theta \quad ; \quad y = r \sin \theta$$

$$\text{met } r = \sqrt{x^2 + y^2} \text{ en } \theta = \text{Bogen } \frac{y}{x}$$

exp. functie in $\mathbb{C}$ -vlak:

$$e^z = e^{x+iy} = e^x (\cos y + i \sin y)$$

$$\Rightarrow z = re^{i\theta} = r (\cos \theta + i \sin \theta)$$

$$\leadsto z^n = r^n (\cos(n\theta) + i \sin(n\theta))$$

H4

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \frac{1}{\cos^2 x}$$

$$(\text{Bogen } x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\text{Bogen } x)' = \frac{1}{1+x^2}$$

$$(c)' = 0$$

$$(x^n)' = n \cdot x^{n-1}$$

$$\begin{array}{|l} (f \cdot g)' \\ = \\ f' \cdot g + f \cdot g' \end{array}$$

$$(a^x)' = a^x \ln a$$

$$(\ln x)' = 1/x$$

$$(\log_a x)' = \frac{1}{x} \ln a$$

$$(e^x)' = e^x$$

$$(x f)' = x \cdot f'$$

$$(f \pm g)' = f' \pm g'$$

$$(f^n)' = n \cdot f^{n-1} f'$$

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$$



115

vgl raaklijn : $y - f(x_0) = f'(x_0) (x - x_0)$

speciale limiet : $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$

$$\text{bv. } * \lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \lim_{x \rightarrow 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x \rightarrow 2} (x+2) = 4$$

$$* \lim_{x \rightarrow \infty} \frac{2x^2 + 5}{x^2 + 2x} \stackrel{\left(\frac{\infty}{\infty}\right)}{=} \lim_{x \rightarrow \infty} \frac{2 + 5/x^2}{1 + 2/x} = 2$$

$$* \lim_{x \rightarrow +\infty} (\sqrt{x^2 + x} - \sqrt{x^2 - 1}) \stackrel{\left(\frac{\infty}{\infty}\right)}{=} \lim_{x \rightarrow +\infty} \left[\sqrt{x^2 + x} - \sqrt{x^2 - 1} \right] \frac{\sqrt{x^2 + x} + \sqrt{x^2 - 1}}{\sqrt{x^2 + x} + \sqrt{x^2 - 1}}$$

$$= \lim_{x \rightarrow +\infty} \frac{(x^2 + x) - (x^2 - 1)}{\sqrt{x^2 + x} + \sqrt{x^2 - 1}} = \lim_{x \rightarrow +\infty} \frac{x + 1}{\sqrt{x^2 + x} + \sqrt{x^2 - 1}}$$

$$\stackrel{\left(\frac{\infty}{\infty}\right)}{=} \stackrel{\left(\frac{1}{x}\right)}{=} \lim_{x \rightarrow +\infty} \frac{1 + 1/x}{\sqrt{1 + 1/x} + \sqrt{1 - 1/x^2}} = \frac{1}{1+1} = \frac{1}{2}$$

$$* f(x) = \begin{cases} x+2 & \text{als } x \leq 1 \\ -\frac{1}{2}x + c & \text{als } x > 1 \end{cases}$$

$$\rightarrow \lim_{x \rightarrow 1-} f(x) = \lim_{x \rightarrow 1-} (x+2) = 3$$

$$\rightarrow \lim_{x \rightarrow 1+} f(x) = \lim_{x \rightarrow 1+} \left(-\frac{1}{2}x + c\right) = -\frac{1}{2} + c$$

[H5]

$$f(x) = \sqrt{1+x^2}, [0, \sqrt{8}]$$

$$* f'(x) = \frac{2x}{2\sqrt{1+x^2}} = \frac{x}{\sqrt{1+x^2}}$$

$$* f'(x) = 0 \Leftrightarrow x = 0$$

* TS:

x	0	$\sqrt{8}$
$f'(x)$	0	+
$f(x)$	1	3

* stationair punt: $x = 0$

absoluit min: $x = 0$

absoluit max: $x = \sqrt{8}$

[H6]

$$\int dx = x + C$$

$$\int x^m dx = \frac{x^{m+1}}{m+1} + C$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \frac{1}{\cos^2 x} dx = \tan x + C$$

$$\int \frac{1}{\sqrt{x^2-1}} dx = \operatorname{Bogsin} x + C$$

$$\int \frac{1}{x^2+1} dx = \operatorname{Bogtan} x + C$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln|ax+b| + C$$



H6

Als f even is, dan $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$

Als f oneven is, dan $\int_{-a}^a f(x) dx = 0$

oneigenlijke integralen:

$$\int_a^{\infty} f(x) dx = \lim_{b \rightarrow \infty} \int_a^b f(x) dx$$

H7

oppervlakte:

→ Cart. coörd: $S = \int_a^b f(x) dx$

→ polcoörd: $\frac{1}{2} \int_a^b [f(\theta)]^2 d\theta$

lengte v.e. kromme:

→ Cart. coörd: $\int_a^b \sqrt{1 + [f'(x)]^2} dx$

→ par. vgl: $\int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2} dt \left\{ \begin{array}{l} x = x(t) \\ y = y(t) \end{array} \right.$

→ polcoörd: $\int_{\theta_1}^{\theta_2} \sqrt{[f(\theta)]^2 + [f'(\theta)]^2} d\theta \left\{ \begin{array}{l} x = f(\theta) \cos \theta \\ y = f(\theta) \sin \theta \end{array} \right.$

omwentelingslichaam volume b

→ rond x-as ? $V = \pi \int_a^b [f(x)]^2 dx$

[118]

partiele integratie : $\int f \cdot g' = f \cdot g - \int f' \cdot g$

rationale / irrationale functies

rationale goniometrische integralen :

$$x = \tan \frac{\theta}{2} ; \theta = 2 \arctan x ; d\theta = \frac{2}{1+x^2} dx$$

[119]

$$\text{grad } f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$$

stelling v. Schwarz & Young : $\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y}$

raakvlak grafiek :

$$z - z_0 = a(x - x_0) + b(y - y_0)$$

$$\text{met } a = \left(\frac{\partial z}{\partial x} \right) (x_0, y_0)$$

$$b = \left(\frac{\partial z}{\partial y} \right) (x_0, y_0)$$

richtingsafgeleide : $D_\theta f = \cos \theta \frac{\partial z}{\partial x} + \sin \theta \frac{\partial z}{\partial y}$



H10

scheidbare veranderlijken:

$$\frac{dx}{dt} = f(x) g(t) \Leftrightarrow \int \frac{1}{f(x)} dx = \int g(t) dt$$

lin. DV v. 1^e orde:

→ homogene DV

→ variatie v. cten

H11

de coëff.:

$$x_{1,2}(t) = e^{\lambda_{1,2} t} \rightsquigarrow \text{karakt. vgl: } \lambda^2 + a\lambda + b = 0$$

niet-homogene lin. DV.

$$x_H(t) = c_1 x_1(t) + c_2 x_2(t)$$

$$x(t) = x_H(t) + x_p(t)$$

$$\text{met } x_p(t): \frac{d^2 x_p}{dt^2} + a \frac{dx_p}{dt} + b x_p = r(t)$$

methode
v. onbepaalde
coëff.

H 12

stationaire punt: $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0$ (in (a, b))

$$\Rightarrow \text{grad } f = (0, 0)$$

bv: $f(x, y) = x^3 + 6xy^2 - 2y^3 - 12x$

$$\rightarrow \frac{\partial f}{\partial x} = 3x^2 + 6y^2 - 12$$

$$\rightarrow \frac{\partial f}{\partial y} = 12xy - 6y^2$$

$$\rightarrow \text{stat. punten: } \begin{cases} 3x^2 + 6y^2 - 12 = 0 \\ 12xy - 6y^2 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} 3x^2 + 6y^2 - 12 = 0 \\ 6y(2x - y) = 0 \end{cases}$$

Dus: $y = 0$ of $y = 2x$

* $y = 0$: dan $3x^2 - 12 = 0$
 $\Leftrightarrow x = \pm 2$

$\hookrightarrow$ 2 stat. ptn: $(2, 0)$ en $(-2, 0)$

* $y = 2x$: dan $3x^2 + 6(2x)^2 - 12 = 0$
 $\Leftrightarrow 3x^2 + 24x^2 - 12 = 0$
 $\Leftrightarrow 27x^2 = 12$

$$\Leftrightarrow x = \pm \frac{2}{3}, \text{ dus } y = \pm \frac{4}{3}$$

$\hookrightarrow$ 2 stat. ptn: $(\frac{2}{3}, \frac{4}{3})$ en $(-\frac{2}{3}, -\frac{4}{3})$

total = 4 stat. ptn.

extrema / zadelpunt:

$$\Delta = \left(\frac{\partial^2 f}{\partial x^2} \right) \left(\frac{\partial^2 f}{\partial y^2} \right) - \left(\frac{\partial^2 f}{\partial x \partial y} \right)^2$$

→ $\Delta < 0$; dan (a, b) = zadelpunt stat. ptn

→ $\Delta > 0$ en $\frac{\partial^2 f}{\partial x^2} > 0$; dan bereikt f in (a, b) een lokaal min.

→ $\Delta > 0$ en $\frac{\partial^2 f}{\partial x^2} < 0$; dan bereikt f in (a, b) een lokaal max.

Vb. $f(x, y) = x^3 + 6xy^2 - 2y^3 - 12x$

→ stat. punten: $(2, 0); (-2, 0); (\frac{2}{3}, \frac{4}{3}); (-\frac{2}{3}, \frac{4}{3})$

$$\rightarrow \frac{\partial f}{\partial x} = 3x^2 + 6y - 12$$

$$\Rightarrow \frac{\partial^2 f}{\partial x^2} = 6x$$

$$\rightarrow \frac{\partial f}{\partial y} = 12xy - 6y^2$$

$$\Rightarrow \frac{\partial^2 f}{\partial xy^2} = 12x - 12y$$

$$\rightarrow \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial f}{\partial x} \left(\frac{\partial f}{\partial y} \right) = 12y$$

→ $12xy - 6y^2$ afleiden wrt x .

* in $(2, 0)$:

$$\left. \begin{aligned} \rightarrow \frac{\partial^2 f}{\partial x^2} &= 12 \\ \rightarrow \frac{\partial^2 f}{\partial y^2} &= 24 \\ \rightarrow \frac{\partial^2 f}{\partial x \partial y} &= 0 \end{aligned} \right\} \begin{aligned} \Delta &= 12 \cdot 24 - 0 > 0 \\ \text{en } \frac{\partial^2 f}{\partial x^2} &> 0 \end{aligned}$$

$\Rightarrow$ functie bereikt in $(2, 0)$ lok. min

* in $(-2, 0)$:

$$\left. \begin{aligned} \rightarrow \frac{\partial^2 f}{\partial x^2} &= -12 \\ \rightarrow \frac{\partial^2 f}{\partial y^2} &= -24 \\ \rightarrow \frac{\partial^2 f}{\partial x \partial y} &= 0 \end{aligned} \right\} \begin{aligned} \Delta &= (-12) \cdot (-24) - 0 > 0 \\ \text{en } \frac{\partial^2 f}{\partial x^2} &< 0 \end{aligned}$$

$\Rightarrow$ functie bereikt in $(-2, 0)$ lok. max.

lagrange techniek: hiermee ik we stationaire punten bekomen die kandidaat-extrema zijn onder bepaalde voorwaarden.

vb: $f(x, y) = 4x^2 - 4xy + y^2$

onder vw'en $g(x, y) = x^2 + y^2 = 20$

↳ straal cirkel
= $\sqrt{20}$

$$\rightarrow \frac{\partial f}{\partial x} = 8x - 4y$$

$$\frac{\partial f}{\partial y} = -4x + 2y$$

$$\rightarrow \frac{\partial g}{\partial x} = 2x$$

$$\frac{\partial g}{\partial y} = 2y$$

→ stat. ptr volden aan:

$$\begin{cases} 8x - 4y = 2\lambda x \\ -4x + 2y = 2\lambda y \\ x^2 + y^2 = 20 \end{cases}$$

$$\Leftrightarrow \lambda = \frac{4x - 2y}{x} = \frac{-2x + y}{y}$$

$$\Leftrightarrow 2x^2 + 3xy - 2y^2 = (2x - y)(x + 2y) = 0$$

dan: $y = 2x$ of $x = -2y$

* Als $y = 2x$, dan

$$x^2 + y^2 = 20 \Rightarrow 5x^2 = 20$$

$$\Leftrightarrow x = \pm 2$$

↳ stat. pnt : $(2, 4)$ dan $(-2, -4)$

* Als $x = -2y$, dan

$$x^2 + y^2 = 20 \Rightarrow 5y^2 = 20$$

$$\Leftrightarrow y = \pm 2$$

↳ stat. pnt : $(-4, 2)$ dan $(4, -2)$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$f'(x) = \frac{1}{2x^7} - 5 \left(\frac{\ln \sqrt{x}}{x^5} \right)$$

OK

OK

$$f(x) = \frac{\ln \sqrt{x}}{x^5} \quad \text{voor } x > 0$$

a) Taylorveelterm $P_2(x)$ v. graad 2 rond $x=1$

$$\rightarrow f(x) = \frac{\ln \sqrt{x}}{x^5}$$

$$\hookrightarrow f(1) = \frac{\ln 1}{1} = 0$$

$$\begin{aligned} \rightarrow f'(x) &= \frac{(\ln \sqrt{x})' \cdot x^5 - \ln \sqrt{x} \cdot 5x^4}{x^{10}} \\ &= \frac{\left(\frac{1}{2\sqrt{x}} \cdot \frac{1}{\sqrt{x}}\right) \cdot x^5 - \ln \sqrt{x} \cdot 5x^4}{x^{10}} \\ &= \frac{\frac{1}{2x^2} \cdot x^5 - \ln \sqrt{x} \cdot 5x^4}{x^{10}} = \frac{x^3 \left(\frac{1}{2} - \ln \sqrt{x} \cdot 5x\right)}{x^{10}} \\ &= \frac{1 - 10 \ln \sqrt{x} \cdot x}{2x^7} = \frac{1}{2x^7} - 5 \frac{\ln \sqrt{x}}{x^6} \end{aligned}$$

$$\hookrightarrow f'(1) = \frac{1}{2} - 5 \frac{\ln 1}{1} = \frac{1}{2}$$

$$\rightarrow f''(x) = \frac{-7}{2x^8} - 5 \cdot \frac{\frac{1}{2x^2} \cdot x^{84} - \ln \sqrt{x} \cdot 6x^5}{x^{12}}$$

$$= \frac{-7}{2} \cdot \frac{1}{x^8} - 5 \cdot \frac{x^4 \left(\frac{1}{2} - 6 \ln \sqrt{x} \cdot x \right)}{x^{12}}$$

$$= \frac{-7}{2} \cdot \frac{1}{x^8} - 5 \cdot \left(\frac{1 - 12x \ln \sqrt{x}}{2x^8} \right)$$

$$= \frac{-7}{2} \cdot \frac{1}{x^8} - 5 \cdot \left(\frac{1}{2x^8} - \frac{6 \ln \sqrt{x}}{x^8} \right)$$

$$= \frac{-7}{2} \cdot \frac{1}{x^8} - \frac{5}{2x^8} + \frac{30 \ln \sqrt{x}}{x^8}$$

$$\hookrightarrow f''(1) = \frac{-7}{2} - \frac{5}{2} = -6$$

$$P_2(x) = \frac{1}{2}x - 3x^2$$

b) rechte π raaklijn aan f in punt $x = 1$

MEENETEN EXAMEN WISKUNDE

- * mondmarker
- * studentenkaart
- * GRM
- * cursus Wisk I & II
- * water + eten
- * schrijfgerei

Oef. 1.22 $P(n) = 1 + 3 + 5 + \dots + (2n-1) = n^2$
 met $n \in \mathbb{N}_0$

① Basisstap voor $P(1)$:

LL/RL: $1 \neq 1 \Rightarrow$ de bewering $P(1)$ is waar
 dus basisstap is bewezen

② Inductiestap:

* Inductiehypothese:

veronderstel $P(k)$ is waar (met $k \in \mathbb{N}_0$):

$$1 + 3 + 5 + \dots + (2k-1) = k^2$$

* TB:

$P(k+1)$ is waar:

$$1 + 3 + 5 + \dots + (2(k+1)-1) = (k+1)^2$$

③ Bewijs:

* LL: $1 + 3 + 5 + \dots + (2(k+1)-1)$
 $= 1 + 3 + 5 + \dots + (2k+2-1)$
 $= 1 + 3 + 5 + \dots + (2k+1)$

* RL: $(k+1)^2 = k^2 + 2k + 1$

$$\Rightarrow \underbrace{1 + 3 + 5 + \dots + (2k+1)}_{k^2 + 2k + 1} \stackrel{!}{=} k^2 + 2k + 1$$

④ conclusie:

Hk

af. 2.25

$$\frac{x^2 + 2}{x^2 - 1} \Rightarrow \text{partieelbreuken}$$

$$\frac{x^2 + 2}{x^2 - 1} \rightarrow \frac{A}{x+1} + \frac{B}{x-1}$$

$$\Leftrightarrow x^2 + 2 = A(x-1) + B(x+1)$$

$$\Leftrightarrow x^2 + 2 = Ax - A + Bx + B$$

$$\Leftrightarrow \begin{cases} 1 = 0 \\ 0 = A + B \\ 2 = -A + B \end{cases} \quad ??$$

$$\frac{x^2 + 2}{x^2 - 1} = \frac{x^2 - 1 + 3}{x^2 - 1} = \frac{x^2 - 1}{x^2 - 1} + \frac{3}{x^2 - 1}$$

$$= 1 + \frac{3}{x^2 - 1} \xrightarrow{\text{PB}} 3 = \frac{A}{x+1} + \frac{B}{x-1}$$

$$\Leftrightarrow 3 = Ax - A + Bx + B$$

$$\Leftrightarrow \begin{cases} 0 = A + B \\ 3 = -A + B \end{cases} \Leftrightarrow \begin{cases} B = -A \\ 3 = -A - A \end{cases}$$

$$\Leftrightarrow \begin{cases} B = -3/2 \\ A = -3/2 \end{cases}$$

vb. 3.8.3 :

geef alle opl. v. $e^z = -2$.

* we schrijven $z = x + iy$,

dan is $|e^z| = e^x$ en $\arg e^z = y$

* Omdat geldt $|-2| = 2$ en $\arg(-2) =$

$\pi + 2k\pi$
 $\hookrightarrow$ we zien dat aan $e^z = -2$ voldaan is

als en slechts als: $e^z = 2$ en

$y = \pi + 2k\pi$ met $k \in \mathbb{Z}$

* dus $x = \ln 2$ en $y = \pi + 2k\pi$ met $k \in \mathbb{Z}$

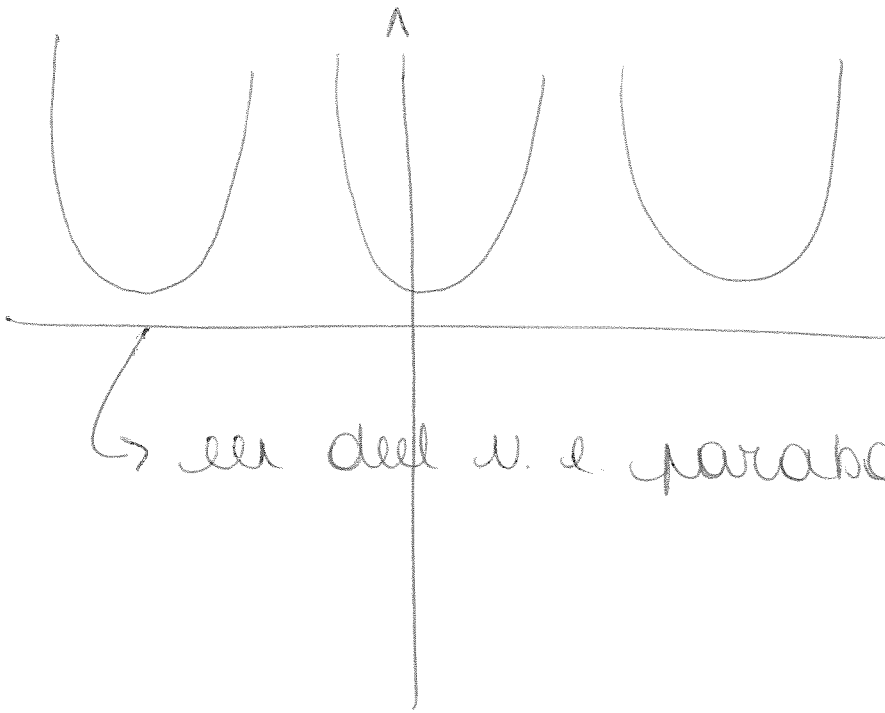
$\hookrightarrow$ er zijn dus ∞ veel opl.

oef. 3.54

$$K_c : \rho = \frac{4}{c + \cos \theta} \quad ; \quad -\frac{\pi}{4} < \theta < \frac{\pi}{4}$$

→ schets kromme voor $c = 1$; wat voor soort kromme is K_c voor $c = 1$

$$* K_c : \rho = \frac{4}{1 + \cos \theta}$$



→ een deel v. e. parabool

oef. 4.35

$$y^2 = x^3 - px + p \quad \text{met } p \in \mathbb{R}$$

a) stel raaklijn op aan kromme in $P: (1, 1)$

$$\Rightarrow y = \sqrt{x^3 - px + p} = f(x)$$

$$* f'(x) = \frac{1}{2} \cdot \frac{1}{\sqrt{x^3 - px + p}} \cdot (3x^2 - p)$$

$$* f'(1) = \frac{1}{2} \cdot \frac{1}{\sqrt{1 - p + p}} \cdot (3 \cdot 1 - p)$$

$$= \frac{1}{2} \cdot (3 - p)$$

$$* \text{vgl raaklijn: } y - f(x_0) = f'(x_0)(x - x_0)$$

$$\Rightarrow y - 1 = \frac{3-p}{2} (x - 1)$$

$$\Leftrightarrow y = \frac{3-p}{2} x - \frac{3-p}{2} + 1$$

$$\Leftrightarrow y = \frac{3-p}{2} x + \frac{p-1}{2}$$

$$= \left(-\frac{7}{4}, -\frac{3}{8}\right) \leftarrow Q: ?$$

b) neem $p = 2$; raaklijn snijdt kromme in ~~100~~

$$* \Rightarrow y = \frac{3-p}{2} x + \frac{p-1}{2}$$

$$\Leftrightarrow y = \frac{1}{2} x + \frac{1}{2} \quad (\text{raaklijn})$$

$$* \text{ kromme: } y^2 = x^3 - px + p$$

$$\Leftrightarrow y = \sqrt{x^3 - 2x + 2}$$

* waar snijdt raaklijn kromme?

$\hookrightarrow$ oplossen.

$$\frac{1}{2} x + \frac{1}{2} = \sqrt{x^3 - 2x + 2}$$

$$\text{BVW: } x^3 - 2x + 2 \geq 0$$

$$\Leftrightarrow \left(\frac{1}{2} x + \frac{1}{2}\right)^2 = x^3 - 2x + 2$$

$$\Leftrightarrow \frac{1}{4} x^2 + \frac{1}{2} x + \frac{1}{4} = x^3 - 2x + 2$$

$$\Leftrightarrow 0 = x^3 - \frac{1}{4} x^2 - \frac{5}{2} x + \frac{7}{4}$$

$$\Leftrightarrow 0 = (x-1) \left(x^2 + \frac{3}{4}x - \frac{7}{4}\right)$$

~~$\Leftrightarrow 0 \neq$~~

1	$-\frac{1}{4}$	$-\frac{5}{2}$	$\frac{7}{4}$
1	1	$\frac{3}{4}$	$-\frac{7}{4}$
1	$\frac{3}{4}$	$-\frac{7}{4}$	0

alle leden $\times 4$
 $\Leftrightarrow 0 = 4x^3 - x^2 - 10x + 7 = 0$

$$\Leftrightarrow (x-1) \cdot (GRH): x = 1$$

$$Q = \left(-\frac{7}{4}, -\frac{3}{8}\right)$$

$$x = -\frac{7}{4}$$

oef. 5.28

Taylorveelterm v. $f(x) = \frac{1}{ce^x - 1}$ rond $x=0$
graad 2 ($c > 0$). ~~$\frac{1}{e}$~~ ~~$\frac{1}{e^x - 1}$~~

* Taylorveelterm rond $x=0$ en graad 2

$$P_n = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 = \frac{1}{c-1} + \frac{c}{(c-1)^2}x + \frac{c(c+1)}{2(c-1)^3}x^2$$

$$(*) \Rightarrow f(0) = \frac{1}{c \cdot e^0 - 1} = \frac{1}{c-1}$$

$$(*) \Rightarrow f'(x) = \frac{-ce^x}{(ce^x - 1)^2}$$

$$\Rightarrow f'(0) = \frac{-c}{(ce^0 - 1)^2} = \frac{-c}{(c-1)^2}$$

$$\begin{aligned} & 2ce^x(ce^x - 1)^{-3} \cdot ce^x \\ & - ce^x(ce^x - 1)^{-2} \\ & = 2c^2e^{2x}(ce^x - 1)^{-3} - ce^x(ce^x - 1)^{-2} \end{aligned}$$

$$(*) \Rightarrow f''(x) = \frac{ce^x \cdot (ce^x - 1)^2 - ce^x (2 \cdot (ce^x - 1) \cdot e^x)}{(ce^x - 1)^4}$$

$$= \frac{ce^x \cdot (ce^x - 1)^2 - 2 \cdot ce^{2x} (ce^x - 1)}{(ce^x - 1)^4}$$

$$\Rightarrow f''(0) = \frac{c(c+1)}{2(c-1)^3} \quad [2c^2(c-1)^{-3} - c(c-1)^{-2}]x^2$$

def. 7.32

staaf met lengte $l > 1$ heeft $\rho(x) = 3x^{-1/2} |x-1|$
met $0 < x < l$ en tot massa $M = 8$.

a) $l = ?$

$$M = \int_0^l 3x^{-1/2} |x-1| dx \quad \text{met } l > 1$$

$$M = \int_0^1 3x^{-1/2} (-x+1) dx + \int_1^l 3x^{-1/2} (x-1) dx$$

$$M = \int_0^1 (-3x^{-1/2} \cdot x + 3x^{-1/2}) dx + \int_1^l (3x^{-1/2} \cdot x - 3x^{-1/2}) dx$$

$$M = -3 \int_0^1 x^{1/2} dx + 3 \int_0^1 x^{-1/2} dx + 3 \int_1^l x^{1/2} dx - 3 \int_1^l x^{-1/2} dx$$

$$M = -3 \cdot \frac{2}{3} [x^{3/2}]_0^1 + 3 \cdot 2 [x^{1/2}]_0^1 + 3 \cdot \frac{2}{3} [x^{3/2}]_1^l - 6 [x^{1/2}]_1^l$$

$$M = -2(1-0) + 6(1-0) + 2(\sqrt{l^3} - 1) - 6(\sqrt{l} - 1)$$

$$M = -2 + 6 + 2\sqrt{l^3} - 2 - 6\sqrt{l} + 6$$

$$\Rightarrow 0 = 2\sqrt{l^3} - 6\sqrt{l}$$

$$0 = 2l\sqrt{l} - 6\sqrt{l}$$

$$0 = 2\sqrt{l} (l-3)$$

$$0 = 2\sqrt{l} \quad \text{of} \quad l-3=0$$

of ~~$l=0$~~

want $l > 0$

$$\Rightarrow \boxed{l=3}$$

vervalg af 7.32

$$b) X = \frac{1}{\pi} \int_0^1 x \cdot 3x^{-1/2} |x-1| dx$$

$$X = \int_0^1 \frac{3}{8} x^{1/2} |x-1| dx$$

$$X = \int_0^3 \frac{3}{8} x^{1/2} |x-1| dx$$

$$c) f(x) = \frac{3}{8} x^{1/2} |x-1|$$

$$n=4 \text{ en } [0, 3]$$

j	x_j	$f(x_j)$
0	0	$\frac{0}{3\sqrt{3}}$
1	$3/4$	$\frac{3\sqrt{3}}{64}$
2	$3/2$	$\frac{3\sqrt{6}}{32}$
3	$9/4$	$\frac{45}{64}$
4	3	$\frac{3\sqrt{3}}{4}$

$$\overline{T}_4 = \frac{3 \cdot 0}{8}$$

$$\left[0 + \frac{6\sqrt{3}}{64} + \frac{6\sqrt{6}}{32} + 2 \cdot \frac{45}{64} + \frac{3\sqrt{3}}{4} \right]$$

$$= 1,247605198$$

Q. 8.28

$$a) \int_0^{\infty} |x-c| e^{-2x} dx$$

$$c > 0$$

$$\Rightarrow \int_0^c (-x+c) e^{-2x} dx + \int_c^{\infty} (x-c) e^{-2x} dx$$

part.) $u = -x+c \Rightarrow du = -dx$ | $u = x-c \Rightarrow du = dx$
 $dv = e^{-2x} dx \Rightarrow v = -\frac{1}{2} e^{-2x} + C$ | $dv = e^{-2x} dx \Rightarrow v = -\frac{1}{2} e^{-2x} + C$

$$= \left[-\frac{1}{2} (-x+c) e^{-2x} - \frac{1}{2} \int e^{-2x} dx \right]_0^c + \left[-\frac{1}{2} (x-c) e^{-2x} + \frac{1}{2} \int e^{-2x} dx \right]_c^{\infty}$$

$$= \left[-\frac{1}{2} (-x+c) e^{-2x} + \frac{1}{4} e^{-2x} \right]_0^c + \left[-\frac{1}{2} (x-c) e^{-2x} - \frac{1}{4} e^{-2x} \right]_c^{\infty}$$

$$= \left[0 + \frac{1}{4} e^{-2c} - \left(-\frac{c}{2} + \frac{1}{4} \right) \right] + \left[0 + 0 - \left(0 - \frac{1}{4} e^{-2c} \right) \right]$$

$$= \frac{1}{4} e^{-2c} + \frac{c}{2} - \frac{1}{4} + \frac{1}{4} e^{-2c}$$

$$= \frac{1}{2} e^{-2c} + \frac{c}{2} - \frac{1}{4}$$

$$C < 0$$

$$\int_0^{\infty} (x-c) e^{-2x} dx$$

$$\begin{cases} u = x - c \Rightarrow du = dx \end{cases}$$

$$\begin{cases} dv = e^{-2x} dx \Rightarrow v = -\frac{1}{2} e^{-2x} \end{cases}$$

$$= \left[-\frac{1}{2} e^{-2x} (x-c) + \frac{1}{2} \int e^{-2x} dx \right]_0^{\infty}$$

$$= \left[-\frac{1}{2} e^{-2x} (x-c) - \frac{1}{4} e^{-2x} \right]_0^{\infty}$$

$$= \left[0 + 0 - \left(\frac{c}{2} - \frac{1}{4} \right) \right] = \frac{1}{4} - \frac{c}{2}$$

Ques 24

$$f(x, y) = xy e^{-x^2 - y^2}$$

$$\begin{aligned} * \frac{\partial f}{\partial x} &= y \cdot e^{-x^2 - y^2} + xy e^{-x^2 - y^2} (-2x) \\ &= y \cdot e^{-x^2 - y^2} - 2x^2 y e^{-x^2 - y^2} \\ &= y e^{-x^2 - y^2} (1 - 2x^2) \end{aligned}$$

$$\begin{aligned} * \frac{\partial f}{\partial y} &= x \cdot e^{-x^2 - y^2} + xy e^{-x^2 - y^2} (-2y) \\ &= x e^{-x^2 - y^2} - 2xy^2 e^{-x^2 - y^2} \\ &= x e^{-x^2 - y^2} (1 - 2y^2) \end{aligned}$$

$$\begin{aligned} * \frac{\partial f}{\partial x} = 0: \quad 0 &= y e^{-x^2 - y^2} (1 - 2x^2) \\ 0 &= 1 - 2x^2 \end{aligned}$$

$$x = \pm \sqrt{\frac{1}{2}}$$

$$* \frac{\partial f}{\partial y} = 0: \quad 0 = x e^{-x^2 - y^2} (1 - 2y^2)$$

$$0 = 1 - 2y^2$$

$$y = \pm \sqrt{\frac{1}{2}}$$

Ptn: $(\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}}), (-\sqrt{\frac{1}{2}}, -\sqrt{\frac{1}{2}}), (-\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}}), (\sqrt{\frac{1}{2}}, -\sqrt{\frac{1}{2}})$ and $(0, 0)$

oef. 10.21

a) $a'(t) = -2k a^2(t)$ met $a(0) = a(0)$

$$\frac{da}{dt} = -2k a^2(t)$$

$$\int \frac{da}{a^2(t)} = -2k \int dt$$

$$\frac{-1}{a(t)} = -2kt + c$$

duz: $a'(t) = \frac{1}{2kt + c}$

[c] $a(0) = \frac{1}{2k \cdot 0 + c}$ dus $c = \frac{1}{a(0)}$

[dus] $a'(t) = \frac{1}{2kt + \frac{1}{a(0)}}$

$$a^*(t) = \frac{1}{\frac{2kt(a(0) + 1)}{a(0)}}$$

$$a(t) = \frac{a(0)}{2kt \cdot a(0) + 1}$$

vervolg af w. 21

$$b) * \frac{1}{2} a(0) = \frac{a(0)}{2k a(0) + 1}$$

$$\frac{1}{2} a(0) = \frac{a(0)}{20k a(0) + 1}$$

$$\frac{1}{2} a(0) [20k a(0) + 1] = a(0)$$

$$10a(0)k + \frac{1}{2} a(0) = 1 \cdot a(0)$$

$$10a(0)k = 1 - \frac{1}{2}$$

$$k = \frac{1}{20a(0)}$$

$$* \frac{1}{10} a(t) = \frac{a(t)}{2 \cdot \frac{1}{20a(t)} \cdot t a(t) + 1}$$

$$\frac{1}{10} a(t) = \frac{a(t)}{\frac{1}{10} t + 1}$$

$$\frac{1}{10} \left(\frac{1}{10} t + 1 \right) = 1 \Leftrightarrow \frac{1}{100} t + \frac{1}{10} = 1$$

$$\Leftrightarrow \frac{1}{100} t = \frac{9}{10} \Rightarrow t = 90.$$

Def 11.21

$$a) \frac{d^2 x}{dt^2} + 2 \frac{dx}{dt} + 2x = \textcircled{1}$$

* Homogene oplossing:

$$\lambda^2 + 2\lambda + 2 = 0$$

$$D = 2^2 - 4 \cdot 1 \cdot 2 = 4i^2$$

$$\hookrightarrow \boxed{\lambda_1} = \frac{-2 + 2i}{2} = \boxed{-1 + i}$$

$$\lambda_2 = \frac{-2 - 2i}{2} = -1 - i$$

$$x_H(t) = (c_1 \cdot \cos t + c_2 \cdot \sin t) e^{-t}$$

* Partiellere oplossing:

$$x_p(t) = A_0$$

$$x'_p(t) = 0$$

$$x''_p(t) = 0$$

$$\left\{ \begin{array}{l} 2A_0 = 1 \Rightarrow A_0 = \frac{1}{2} \\ \text{dus } x_p(t) = \frac{1}{2} \end{array} \right.$$

* $x(t) = x_H(t) + x_p(t)$

$$x(t) = (c_1 \cdot \cos t + c_2 \cdot \sin t) e^{-t} + \frac{1}{2}$$

vervolg of 11.21

$$b) \frac{d^2 x}{dt^2} + \frac{2dx}{dt} = \textcircled{1}$$

* Homogene oplossing:

$$\lambda^2 + 2\lambda = 0 \Rightarrow \lambda(\lambda + 2) = 0$$

$$\Rightarrow \lambda = 0 \text{ of } \lambda = -2$$

$$x_H(t) = C_1 + C_2 \cdot e^{-2t}$$

* Particuliere opl:

$x(t) = 1$ maar dit is reeds een deel vd homog. opl dus we proberen:

$$x_p(t) = At; \quad x'_p(t) = A; \quad x''_p(t) = 0$$

$$\text{we krijgen: } 0 + 2A = 1$$

$$\text{dus } A = \frac{1}{2}$$

$$x_p(t) = \frac{1}{2}t$$

* $x(t) = x_H(t) + x_p(t)$:

$$x(t) = C_1 + C_2 \cdot e^{-2t} + \frac{1}{2}t$$

* Beginnw' en: $x(0) = 0$ en $x'(0) = 1$

$$x'(t) = -2C_2 \cdot e^{-2t} + \frac{1}{2}$$

$$\begin{cases} 0 = C_1 + C_2 e^{-2 \cdot 0} + \frac{1}{2} \cdot 0 \\ 1 = -2C_2 \cdot e^{-2 \cdot 0} + \frac{1}{2} \end{cases}$$

$$\Rightarrow \begin{cases} 0 = C_1 + C_2 \Rightarrow C_2 = -C_1 \\ 1 = -2C_2 + \frac{1}{2} \Rightarrow C_2 = -\frac{1}{4} \text{ dus } C_1 = \frac{1}{4} \end{cases}$$

* Eindeutkomst:

$$x(t) = \frac{1}{4} - \frac{1}{4} e^{-2t} + \frac{1}{2} t$$

oef. 12.30

(150)

$$a) d(x, y) = \sqrt{(x-1)^2 + (y-0)^2}$$

$$\text{afstandsform.} = \sqrt{x^2 - 2x + 1 + y^2}$$

Dit minimaliseren k door $x^2 - 2x + 1 + y^2$ te minimaliseren.

we krijgen dan:

$$f(x, y) = x^2 - 2x + 1 + y^2$$

$$NV: \frac{1}{4}x^2 + y^2 = R^2$$

$$\frac{\partial f}{\partial x} = 2x - 2$$

$$\frac{\partial f}{\partial y} = 2y$$

$$\frac{\partial g}{\partial x} = \frac{1}{2}x$$

$$\frac{\partial g}{\partial y} = 2y$$

- Lagrange tech. $\rightarrow$ $\begin{cases} 2x - 2 = \frac{1}{2}x\lambda \\ 2y = 2y\lambda \end{cases} \Rightarrow \boxed{\lambda = 1}$ indien $y \neq 0$

$$\begin{cases} 2x - 2 = \frac{1}{2}x \\ \lambda = 1 \end{cases} : 2x - 2 - \frac{1}{2}x = 0$$

$$\begin{aligned} \frac{3}{2}x &= 2 \\ \boxed{x &= \frac{4}{3}} \end{aligned}$$

Indien: $x = \frac{4}{3}$ dan $y^2 = R^2 - \frac{1}{4}x^2$

$$y = \pm \sqrt{R^2 - \frac{4}{9}}$$

Indien: $y = 0$ dan $\frac{1}{4}x^2 = R^2 \Rightarrow x = \pm 2R$

De 4 stationaire ptn zijn dus:

$$(2R, 0), (-2R, 0), \left(\frac{4}{3}, \sqrt{R^2 - \frac{4}{3}}\right), \left(\frac{4}{3}, -\sqrt{R^2 - \frac{4}{3}}\right)$$

b) welk pnt op ellips ligt het dichtste bij $(1, 0)$?

afh. v. R

Indien $R \geq \frac{2}{3}$ zijn het de ptn

$$\left(\frac{4}{3}, \sqrt{R^2 - \frac{4}{3}}\right) \text{ en } \left(\frac{4}{3}, -\sqrt{R^2 - \frac{4}{3}}\right)$$

Indien $R < \frac{2}{3}$ is het pnt $(2R, 0)$

oef. 12.31

$$a) \int_x^4 (s^4 - s)(s - 2) ds$$

$$= \int_x^4 (s^3 - 2s^2 - s^2 + 2s) ds$$

$$= \int_x^4 (s^3 - 3s^2 + 2s) ds$$

$$= \int_x^4 s^3 ds - 3 \int_x^4 s^2 ds + 2 \int_x^4 s ds$$

$$= \frac{1}{4} [s^4]_x^4 - \frac{3}{3} [s^3]_x^4 + \frac{2}{2} [s^2]_x^4$$

$$= \frac{1}{4} (4^4 - x^4) - [4^3 - x^3] + [4^2 - x^2]$$

vervolg oef 12.31

$$= \boxed{\frac{1}{4} y^4 - \frac{1}{4} x^4 - y^3 + x^3 + y^2 - x^2} = f(x, y)$$

* we zoeken de stat. ptn v. $f(x, y)$

$$\frac{\partial f}{\partial x} = -x^3 + 3x^2 - 2x$$

$$\frac{\partial f}{\partial y} = y^3 - 3y^2 + 2y$$

$$\begin{cases} -x^3 + 3x^2 - 2x = 0 \\ y^3 - 3y^2 + 2y = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} -x(x^2 - 3x + 2) = 0 & (1) \\ y(y^2 - 3y + 2) = 0 & (2) \end{cases}$$

dan we vinden: (1): $x = 0$

$$x^2 - 3x + 2 = 0 \begin{cases} x = 1 \\ x = 2 \end{cases}$$

(2): $y = 0$

$$y^2 - 3y + 2 = 0 \begin{cases} y = 1 \\ y = 2 \end{cases}$$

De stat. ptn v. $f(x, y)$ zijn dus:

$(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (2, 2)$

b) beperken tot stat. ptn (x, y) met $x < y$
dus $(0, 1)$ en $(0, 2)$ en $(1, 2)$

$$\frac{\partial^2 f}{\partial x^2} = -3x^2 + 6x - 2$$

$$\frac{\partial^2 f}{\partial y^2} = 3y^2 - 6y + 2$$

$$\frac{\partial}{\partial x} \left(\frac{\partial f}{\partial y} \right) = 0$$

* voor pnt $(0, 1)$ →

$$\Delta = -2 \cdot (-1) - (0)^2 = 2 \text{ en } \frac{\partial^2 f}{\partial x^2} < 0$$

⇒ dus lok. max. in $(0, 1)$

* voor pnt $(0, 2)$:

$$\Delta = -2 \cdot (2) - (0)^2 = -4$$

⇒ dus zadelptn in $(0, 2)$

* voor pnt $(1, 2)$:

$$\Delta = 1 \cdot (2) - (0)^2 = 2 \text{ en } \frac{\partial^2 f}{\partial x^2} > 0$$

⇒ dus lok. min. in $(1, 2)$

303 stell. 12.4.1

H3

$e^{i\theta}/3^e$	0°	30°	45°	60°	90°
θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	/

$$* e^z = e^{x+iy} = e^x (\cos y + i \sin y)$$

$$e^x \cdot e^{iy}$$

$$(e^{i\pi} + 1 = 0)$$

$$* z = r(\cos \theta + i \sin \theta)$$

$$z = r e^{i\theta}$$

~~* $z^n = r^n (\cos(n\theta) + i \sin(n\theta))$~~

$$* z^n = r^n (\cos(n\theta) + i \sin(n\theta))$$

* vgl raaklijn

$$y - f(x_0) = f'(x_0) (x - x_0)$$

Wisk 1

~~af 3.54~~

?

~~af 4.35~~

~~af 5.28~~

?

~~af 7.32~~

af 8.28

~~8.28~~

af 9.24

af 10.21

af 11.21

af 12.30

12.31

af 13.22

	Cartesische coördinaten	parametervergelijking	poolcoördinaten
lengte van de kromme	$(y = f(x))$ $\int_a^b \sqrt{1 + [f'(x)]^2} dx$	$\begin{cases} x = x(t) \\ y = y(t) \end{cases}$ $\int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2} dt$	$x = f(\theta) \cos \theta$ $y = f(\theta) \sin \theta$ $\int_a^b \sqrt{[f'(\theta)]^2 + [f(\theta)]^2} d\theta$
oppervlakte onder/binnen de kromme	$S = \int_a^b f(x) dx$		$\int_a^b \frac{1}{2} [f(\theta)]^2 d\theta$
volume omwentelings-lichaam	rond x-as $\pi \int_a^b [f(x)]^2 dx$		
	rond y-as		

Wiskunde 1 - Oude examen vragen

augustus 2014

Vraag 2

$$\int_b^{\infty} \frac{1}{x(x-a)} dx = a \quad \text{met } a > 0 \text{ en } b > a$$

a) $\lim_{c \rightarrow \infty} \int_b^c \frac{1}{x(x-a)} dx$

↳ splitsen in partitieelbreuken

$$\frac{1}{x(x-a)} = \frac{A}{x} + \frac{B}{x-a}$$

$$\Leftrightarrow 1 = Ax - aA + Bx$$

$$\begin{cases} 0 = A + B \\ 1 = -aA \end{cases} \Leftrightarrow \begin{cases} B = 1/a \\ A = -1/a \end{cases}$$

$$\Rightarrow \lim_{c \rightarrow \infty} \left[\frac{-1}{a} \int_b^c \frac{1}{x} dx + \frac{1}{a} \int_b^c \frac{1}{x-a} dx \right]$$

$$= \frac{-1}{a} \lim_{c \rightarrow \infty} \int_b^c \frac{1}{x} dx + \frac{1}{a} \lim_{c \rightarrow \infty} \int_b^c \frac{1}{x-a} dx$$

$$= \frac{-1}{a} \lim_{c \rightarrow \infty} (\ln|c| - \ln|b|) + \frac{1}{a} \lim_{c \rightarrow \infty} (\ln|c-a| - \ln|b-a|)$$

$$= \frac{-1}{a} \lim_{c \rightarrow \infty} \ln|c| + \frac{1}{a} \lim_{c \rightarrow \infty} \ln|b| + \frac{1}{a} \lim_{c \rightarrow \infty} \ln|c-a| - \frac{1}{a} \lim_{c \rightarrow \infty} \ln|b-a|$$

$$(\text{geg}) a = \frac{1}{a} \left(\lim_{c \rightarrow \infty} (-\ln|c| + \ln|b| + \ln|c-a| - \ln|b-a|) \right)$$

$$\Leftrightarrow a^2 = \lim_{c \rightarrow \infty} (-\ln|c| + \ln|b| + \ln|c-a| - \ln|b-a|)$$

$$\Leftrightarrow a^2 = \cancel{-\infty} + \ln|b| + \cancel{\infty} - \ln|b-a|$$

$$\Leftrightarrow a^2 = \ln|b| - \ln|b-a|$$

$$\Leftrightarrow a^2 = \ln \left| \frac{b}{b-a} \right|$$

$$\Leftrightarrow e^{a^2} = \frac{b}{b-a} \quad \Leftrightarrow e^{a^2}(b-a) = b$$

$$\Leftrightarrow e^{a^2}b - ae^{a^2} = b$$

$$\Leftrightarrow -ae^{a^2} = b - e^{a^2}b \quad \Leftrightarrow -ae^{a^2} = b(1 - e^{a^2})$$

$$\Leftrightarrow \frac{-ae^{a^2}}{1 - e^{a^2}} = b = \frac{ae^{a^2}}{e^{a^2} - 1}$$

b) $\lim_{a \rightarrow 0} ab$ met $b = \frac{ae^{a^2}}{e^{a^2} - 1}$

$$= \lim_{a \rightarrow 0} a \cdot \frac{ae^{a^2}}{e^{a^2} - 1} = \lim_{a \rightarrow 0} \frac{a^2 e^{a^2}}{e^{a^2} - 1} \stackrel{0}{=} 0$$

$$= \lim_{a \rightarrow 0} \frac{2ae^{a^2} + a^2 2ae^{a^2}}{2ae^{a^2}} = \lim_{a \rightarrow 0} \underbrace{\frac{2ae^{a^2}}{2ae^{a^2}}}_{=1} + \lim_{a \rightarrow 0} \underbrace{\frac{a^2 2ae^{a^2}}{2ae^{a^2}}}_{=0}$$

$$= 1$$

januari 2013 B

vraag 2

$$K: r = R(\cos \theta + \sin \theta) \quad ; \quad \theta \in [0, \frac{\pi}{2}]$$

met $R = \text{maxste cte}$

a) $R = 4$

$$r = 4(\cos \theta + \sin \theta)$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$r \cdot r = r^4 (\cos \theta + \sin \theta)$$

$$\Leftrightarrow r^2 = 4 (r \cos \theta + r \sin \theta) \rightarrow \text{met } r \cos \theta = x$$

$$r \sin \theta = y$$

$$\Leftrightarrow r^2 = 4(x + y)$$

$$\Leftrightarrow r^2 = 4x + 4y$$

$\Leftrightarrow$

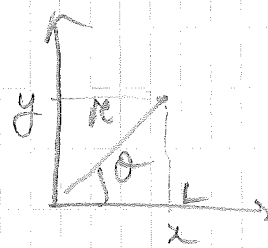
$$x^2 + y^2 - 4x - 4y = 0$$

$$\Leftrightarrow (x-2)^2 + (y-2)^2 = 8$$

$$x^2 - 4x + 4 + y^2 - 4y + 4 = 0$$

$$\Rightarrow \text{middenpunt} = (2, 2)$$

$$\text{straal} = 2\sqrt{2}$$



$$r^2 = x^2 + y^2$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$\frac{R^2}{2} \left(\frac{\pi}{2} + 1 \right) \Leftarrow$ b) opp in het eerste kwadrant $= 0 \leq \theta \leq \frac{\pi}{2}$

$$\text{opp} = \frac{1}{2} \int_0^{\pi/2} (R(\cos \theta + \sin \theta))^2 d\theta$$

$$= \frac{1}{2} \int_0^{\pi/2} R^2 (\cos \theta + \sin \theta)^2 d\theta$$

$$= \frac{1}{2} \int_0^{\pi/2} R^2 (\cos^2 \theta + 2 \cos \theta \sin \theta + \sin^2 \theta) d\theta$$

$$= \frac{R^2}{2} \int_0^{\pi/2} (\underbrace{\cos^2 \theta + \sin^2 \theta}_{=1} + (2 \cos \theta \sin \theta)) d\theta$$

$$= \frac{R^2}{2} \int_0^{\pi/2} (1 + \underbrace{2 \cos \theta \sin \theta}_{= \sin 2\theta}) d\theta$$

$$= \frac{R^2}{2} \int_0^{\pi/2} (1 + \sin 2\theta) d\theta = \frac{R^2}{2} \left[\frac{\theta}{1} + \frac{-1}{2} \cos 2\theta \right]_0^{\pi/2}$$

$$= \frac{R^2}{2} \left(\frac{\pi}{2} - \frac{1}{2} \cos \pi + \frac{1}{2} \cos 0 \right) = \frac{R^2}{2} \left(\frac{\pi}{2} + \frac{1}{2} \right) = \frac{R^2}{2} \left(\frac{\pi}{2} + \frac{1}{2} \right)$$

$$= \frac{R^2}{2} \int_0^{\pi} \sin u + 1 du = \frac{R^2}{2} \left[-\frac{1}{2} \cos u + u \right]_0^{\pi} = \frac{R^2}{2} \left(-\cos \pi + \cos 0 + \pi \right)$$

$$2\theta = u$$

$$2d\theta = du$$

$$\theta = 0 \rightarrow u = 0$$

$$\theta = \frac{\pi}{2} \rightarrow u = \pi$$

Antw.

Jan 2013 (B)

VR 3 (a)

$$(b) x(t) = \frac{t^2 - 4}{4t}$$

VR 1 (a)

$$Q(x) = \frac{3}{4} - \frac{1}{2(x+1)} - \frac{1}{2(x+2)}$$

(b)

Aug 2018

$$a) T_2(x) = 23 - 26x + 11x^2$$

$$b) Q = \left(-\frac{7}{4}, 19\right)$$

02/10/2019

voorbereiding Wiskunde 1 - oefeningen

oefenzitting 1.-2/10/2019

oef. 1.1. (c, f)

$$\begin{aligned} c) \quad \frac{3}{17} : \frac{14}{51} - \frac{7}{6} &= \left(\frac{3}{17} \cdot \frac{51}{14} \right) - \frac{7}{6} = \frac{153}{238} - \frac{7}{6} = \frac{918 - 1666}{1428} \\ &= \frac{-748}{1428} = \frac{-374}{714} = \frac{-187}{357} = \frac{-11}{21} \end{aligned}$$

$$\begin{aligned} f) \quad \frac{1}{m+n} : \left(\frac{1}{m} + \frac{1}{n} \right) &= \frac{1}{m+n} : \left(\frac{n+m}{m \cdot n} \right) \\ &= \frac{1}{m+n} \cdot \frac{m \cdot n}{n+m} = \frac{m \cdot n}{(m+n)^2} \end{aligned}$$

oef. 1.2. (b)

$$b) \quad \sqrt{(\sqrt[3]{a^3})^2} = \left((a^{3/3})^2 \right)^{1/2} = a$$

oef. 1.4 (a, c)

$$a) \quad |x| < 3 \quad \leadsto \quad V = \{-1, -2, 0, 1, 2\}$$

$$c) \quad |x+3| \leq 5 \quad \leadsto \quad V = \{-2, -1, \dots, 8\}$$

oef. 1.6 (a, b)

$$z_1 = 3-i \quad \text{en} \quad z_2 = 1+2i$$

$$a) \quad z_1 + z_2 = 3-i + 1+2i = 4+i$$

$$b) \quad z_1 z_2 = (3-i)(1+2i) = 3 + 6i - i - \underbrace{2i^2}_{+2} = 5 + 5i$$

Def. 1.7(b)

$$z_1 = 2+3i \text{ and } z_2 = -2+i$$

$$b) \overline{z_1 z_2} = 5-5i$$

Def. 1.8(b,c)

$$b) (1-3i)^2 - (1+3i)^2 = 1-6i+9i^2 - (1+6i+9i^2) \\ = -12i \quad (0-12i)$$

$$c) \frac{1-i}{1+i} = \frac{(1-i)(1-i)}{1-i^2} = \frac{1-i-1+i^2}{2} = \frac{-2i}{2} = 0-i$$

Def. 1.18(b)

$$b) \sum_{n=0}^4 \frac{1}{1+n^2} = \frac{1}{1+0^2} + \frac{1}{1+1^2} + \frac{1}{1+2^2} + \frac{1}{1+3^2} + \frac{1}{1+4^2} \\ = \frac{1}{1} + \frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \frac{1}{17} \\ = \frac{9}{5} + \frac{1}{17} = \frac{153+1}{85} = \frac{154}{85}$$

Def. 1.19(b)

$$b) x + x^5 + x^9 + \dots + x^{4k+1} + \dots + x^{41} \\ = \sum_{k=0}^{10} (x^{4k+1})$$

02/10/2019 opexiting 1

✓
0 op. 1.5 (a, c)

a) $|x^2 - 1| \leq 3$

$$-x^2 + 1 \leq 3 \quad \text{ou} \quad 3 \leq x^2 - 1$$

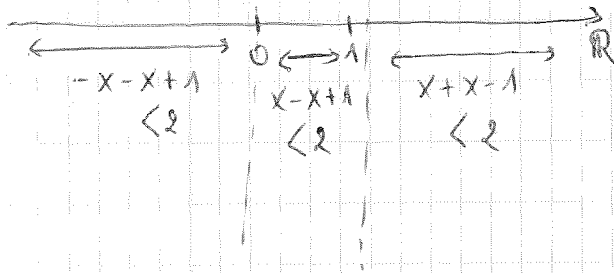
$$x^2 \geq -2 \quad \text{ou} \quad 4 \leq x^2$$

$$x \geq \sqrt{-2} \quad 2 \leq x$$

↓
maximum

$$x \in [-2, 2]$$

c) $|x| + |x-1| < 2$



$$\nearrow x \geq 0 \Rightarrow |x| = x$$

$$\nearrow x < 0 \Rightarrow |x| = -x$$

$$\nearrow x \geq 1 \Rightarrow |x-1| = x-1$$

$$\nearrow x < 1 \Rightarrow |x-1| = -x+1$$

op. 1.7 (c)

$$z_1 = 2+3i \quad \text{ou} \quad z_2 = -2+i$$

$$\begin{aligned} \text{c) } \overline{z_1/z_2} &\Rightarrow z_1/z_2 = \frac{2+3i}{-2+i} = \frac{(2+3i)(-2-i)}{4-i^2} = \frac{-4-2i-6i-3i^2}{5} \\ &= \frac{-1-8i}{5} = \frac{-1}{5} - \frac{8}{5}i \end{aligned}$$

$$\Rightarrow \overline{z_1/z_2} = \frac{-1}{5} + \frac{8}{5}i$$

③=conclusie "Omdat de basisstap ① & de inductiestap ② bewezen zijn, geldt de stelling vanwege het principe v. volledige inductie"

oef. 1.8(f)

$$f) \frac{1}{5} - \frac{3-4i}{3+4i} = \frac{1}{5} - \frac{(3-4i)^2}{9-16i^2} = \frac{1}{5} - \frac{9-24i+16i^2}{25} = \frac{1}{5} - \frac{-7-24i}{25}$$

$$\frac{5}{25} = \frac{1}{5} + \frac{7}{25} + \frac{24i}{25} = \frac{12}{25} + \frac{24i}{25}$$

oef. 1.14(a)

a) coeff v. x^4 in $(2x+5)^{10}$:

$$\binom{10}{6} (2x)^{10-6} 5^6 \Rightarrow \frac{10!}{6!4!} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} = 210$$

$$\Rightarrow 210 \cdot 2^4 x^4 \cdot 5^6 = 52\,500\,000$$

oef. 1.15(b)

$$b) \sum_{k=0}^n (-1)^k \binom{n}{k} = 0 \quad \text{als } n \geq 1$$

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

$$\sum_{k=0}^n \binom{n}{k} \underbrace{a^{n-k}}_1 (-1)^k \leadsto a=1 \text{ en } b=-1$$

$$\Rightarrow a^{n-k} = 1 \Rightarrow n-k=0$$

$$\underline{n=k}$$

7

oef. 1.22

$$n \in \mathbb{N}_0$$

$$= n \geq 1$$

bewijs volledige inductie.

$$1+3+5+\dots+(2n-1)=n^2$$

$$① \underline{n=1} \Rightarrow 1=1^2 \Rightarrow \text{WAAR}$$

$$② (n+1) \Rightarrow 1+3+5+\dots+(2n-1)+(2(n+1)-1) \stackrel{?}{=} (n+1)^2$$

$$1+3+5+\dots+(2n-1)+(2n+1) \stackrel{?}{=} (n+1)^2$$

$$\Rightarrow n^2 + (2n+1)$$

$$n^2 + 2n + 1$$

Def. 1.25 $x > -1$ und $n \in \mathbb{N}$

$$(1+x)^n \geq 1+nx$$

$$1+2x+x^n \geq 1+nx \quad (\Leftrightarrow) \quad x^n + \underbrace{2x-nx}_{(2-n)x} \geq 0$$

① $n=0$ $\Rightarrow x^0 + (2-0)x \geq 0$
 $1+2x \geq 0$ mit $x > -1$
 $\Rightarrow$ klapt

② $n+1$ $\Rightarrow x^{n+1} + (2-(n+1))x \geq 0$
 $x^{n+1} + (1-n)x \geq 0$

① $n=0$ $\Rightarrow (1+x)^0 \geq 1+0x$
 $1 \geq 1$
 $\Rightarrow$ klapt

② $n+1$ $\Rightarrow$ RL: $1+(n+1)x$
 $(\Leftrightarrow) 1+nx+x$

LL: $(1+x)^{n+1}$
 $(\Leftrightarrow) 1+2x+x^{n+1}$

oef. 2.2 (a,c)

a) $\frac{1}{x^2 - 3}$

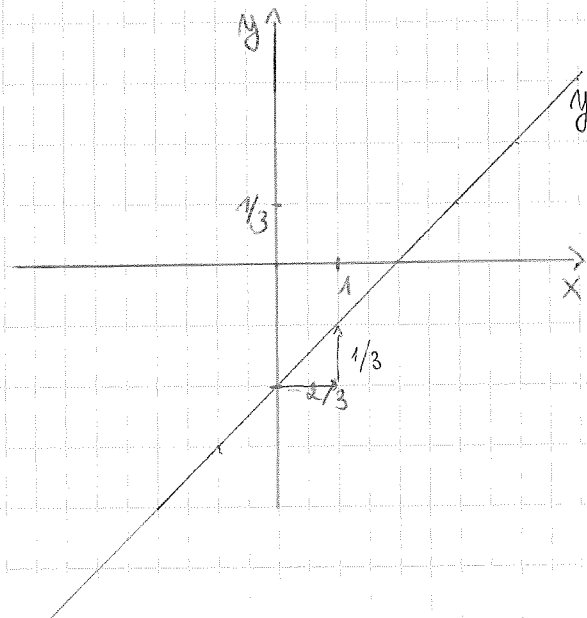
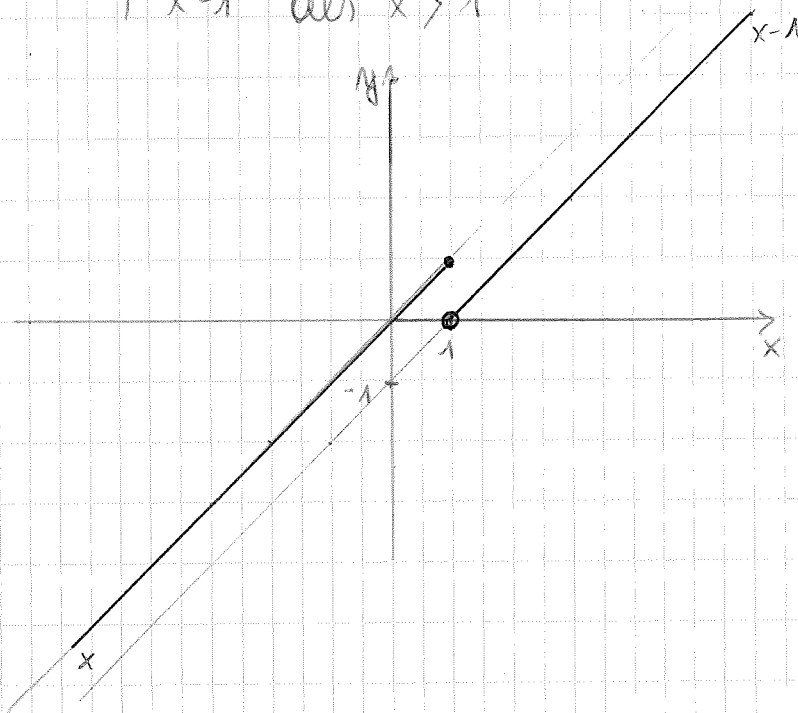
* nulptn $N \Rightarrow x = \pm\sqrt{3}$

* dom = $]-\infty, -\sqrt{3}[\cup]\sqrt{3}, +\infty[$

c) $x + 2|x|$ * dom = $\mathbb{R}$

oef. 2.5 (a)

a) $f(x) = \begin{cases} x & \text{als } x \leq 1 \\ x-1 & \text{als } x > 1 \end{cases}$



oef. 2.8 (b)

b) $x = 3y + 2 \Leftrightarrow \frac{x-2}{3} = y$

$\frac{1}{3}x - \frac{2}{3} = y$

Def. 2.9 (a, c)

$$a) (-1, 1) \text{ en } (2, -4) \Rightarrow \text{rico } a = \frac{-4-1}{2+1} = \frac{-5}{3}$$

$$a \Leftrightarrow y - 1 = \frac{-5}{3} (x + 1)$$

$$y = \frac{-5}{3} x - \frac{5}{3} + 1$$

$$a \Leftrightarrow y = \frac{-5}{3} x - \frac{2}{3}$$

$$c) (2, -1) \text{ en } c \parallel x + y = 3$$

$\hookrightarrow$ derfelde rico $\Rightarrow$ rico $c = -1$

$$c \Leftrightarrow y + 1 = -1 (x - 2)$$

$$y = -x + 2 - 1$$

$$c \Leftrightarrow y = -x + 1$$

Def. 2.11 (a, b)

$$a) x^2 - 3x + 2$$

$$D = 9 - 8 = 1$$

$$x_{1,2} = \frac{3 \pm 1}{2} \begin{matrix} 2 \\ 1 \end{matrix}$$

$$= (x - 2) (x - 1)$$

$$b) x^2 - a^2 = (x - a) (x + a)$$

afz. 2.14 (a)

$$a) y = x - 3 + \frac{1}{x-2} = \frac{(x-3)(x-2) + 1}{x-2} = \frac{x^2 - 5x + 7}{x-2}$$

$$\star \text{ nulpnt } N \Rightarrow x = 2$$

$$\text{nulpnt } T \Rightarrow x^2 - 5x + 7 = 0$$

$$D = 25 - 28 < 0 \quad \checkmark$$

$$\star \underline{VA} : x = 2$$

$$\star \underline{HA} : \quad \checkmark \rightarrow q_T > q_N$$

$$\star \underline{SA} : y = x - 3$$

afz. 2.15 (d)

$$d) y = \frac{1}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3}$$

$$\hookrightarrow 1 = Ax + 3A + Bx - 2B$$

$$\begin{cases} 0 = A + B \\ 1 = 3A - 2B \end{cases} \Leftrightarrow \begin{cases} -A = B \\ \frac{1}{3} + \frac{2}{3}B = A \end{cases}$$

$$\Leftrightarrow \begin{cases} -A = B \\ \frac{1}{3} - \frac{2}{3}A - A = 0 \end{cases} \Leftrightarrow \begin{cases} -A = B \\ \frac{-5}{3}A = -\frac{1}{3} \end{cases}$$

$$\Leftrightarrow \begin{cases} B = -1/5 \\ A = 1/5 \end{cases}$$

$$= \frac{1}{5} \cdot \frac{1}{x-2} - \frac{1}{5} \cdot \frac{1}{x+3}$$

4/10/2019 oefenrilling 2

oef. 2.2. (f)

f) $\frac{\sqrt{x} - \sqrt{x-3}}{x}$

* nulplek $T \Rightarrow \sqrt{x} - \sqrt{x-3} = 0$

$$\begin{cases} x \geq 0 \\ x-3 \geq 0 \end{cases} \Leftrightarrow \begin{cases} x \geq 0 \\ x \geq 3 \end{cases}$$

* nulplek $N \Rightarrow x=0$

$$\text{dom} = x \in [3, +\infty[$$

oef. 2.6 (c)

c) $f(x) = 3x$ en $g(x) = \sqrt{-x}$

* $g \circ f : g(f(x)) = \sqrt{-3x}$ (met $x \leq 0$)

* $f \circ g : f(g(x)) = 3\sqrt{-x}$ (met $x \leq 0$)

oef. 2.11 (a)

a) $x^4 - 5x^2 + 4$

$= (x-1)(x^3 + x^2 - 4x - 4)$

1	0	-5	0	4
1	1	1	-4	-4

$= (x-1)(x+1)(x^2-4)$

1	1	-4	-4	0
---	---	----	----	---

$= (x-1)(x+1)(x-2)(x+2)$

-1	-1	0	4
----	----	---	---

$= (x-1)(x+1)(x-2)(x+2)$

1	0	-4	0
---	---	----	---

Op 2.12.

$$3x^3 - 4x^2 - x + 2 = (x-1)(3x^2 - x - 2) = (x-1)(x-1)(3x+2)$$

$$\begin{array}{r|rrrr} & 3 & -4 & -1 & 2 \\ 1 & & 3 & -1 & -2 \\ \hline & 3 & -1 & -2 & 0 \end{array}$$

* nulpun: $(x-1) \rightarrow x=1$

$$(3x^2 - x - 2) \rightarrow x=1 \text{ en } x = -\frac{2}{3}$$

$$D = 1 + 4 \cdot 3 \cdot 2 = 25$$

$$x_{1,2} = \frac{1 \pm 5}{6} < \frac{1}{3}$$

Op 2.14(c)

$$c) \quad y = \frac{x^2(x-1)}{x^2-4} = \frac{x^2(x-1)}{(x-2)(x+2)} = \frac{x^3-x^2}{x^2-4}$$

* nulpun N: $x=2$ en $x=-2$

* VA: $x=2$

$x=-2$

* HA: geen $\rightarrow q_L T > q_L N$

* SA:

$$\begin{array}{r|l} x^3 - x^2 & x^2 - 4 \\ - (x^3 + 0x^2 - 4x) & x - 1 \\ \hline 0 - x^2 + 4x & \underline{x-1} \\ - (-x^2 + 4x) & = SA \\ \hline 0 & \end{array}$$

$\rightarrow y = x-1$

Def. 2.16 (c)

$$c) \quad y = \frac{x}{(x+1)(x^2+2x+6)}$$

$$\hookrightarrow \frac{A}{x+1} + \frac{Bx+C}{x^2+2x+6}$$

$$x = Ax^2 + 2Ax + 6A + Bx^2 + Bx + Cx + C$$

$$\begin{cases} 0 = A + B \\ 1 = 2A + B + C \\ 0 = 6A + C \end{cases}$$

$$\Leftrightarrow \begin{cases} B = -A \\ 1 = 2A + B + C \\ C = -6A \end{cases}$$

$$\Leftrightarrow \begin{cases} B = -A \\ 1 = 2A - A - 6A \\ C = -6A \end{cases}$$

$$\Leftrightarrow \begin{cases} B = -1/5 \\ -1/5 = A \\ C = +6/5 \end{cases}$$

$$= -\frac{1}{5} \cdot \frac{1}{x+1} + \frac{1}{5} \cdot \frac{x+6}{x^2+2x+6}$$

Def. 2.15 (g)

$$g) \quad y = \frac{x^2(x-1)}{2x^2-4} = \frac{x^2(x-1)}{2(x^2-2)} \stackrel{x^3-x^2}{=} \rightarrow \text{grT} > \text{grN}$$

$$= \frac{1}{2} \cdot \frac{x^3-x^2}{x^2-2}$$

$$\begin{array}{r} x^3 - x^2 - 0x - 0 \quad | \quad x^2 - 2 \\ - x^3 + 0x^2 - 2x \\ \hline 0 - x^2 + 2x \\ - -x^2 + 0x + 2 \\ \hline 0 + 2x - 2 \end{array}$$

$$= \frac{1}{2} \cdot \left(\frac{x-1}{x^2-2} + \frac{(2x-2)}{x^2-2} \right) = \frac{x-1}{2} + \frac{1}{2} \cdot \frac{2x-2}{x^2-2} = \frac{x-1}{2} + \frac{x-1}{x^2-2}$$

$$= \frac{x-1}{2} + \frac{x-1}{x^2-2} = \frac{x-1}{2} + \frac{x-1}{(x-\sqrt{2})(x+\sqrt{2})}$$

$$\hookrightarrow \frac{A}{x-\sqrt{2}} + \frac{B}{x+\sqrt{2}}$$

$$x-1 = Ax + \sqrt{2}A + Bx - \sqrt{2}B$$

$$\begin{cases} 1 = A + B \\ -1 = \sqrt{2}A - \sqrt{2}B \end{cases}$$

$$\begin{cases} 1 = A + B \\ -1 = \sqrt{2}A - \sqrt{2}B \end{cases}$$

$$\Leftrightarrow \begin{cases} B = 1 - A \\ -1 = \sqrt{2}A - \sqrt{2} + \sqrt{2}A \end{cases}$$

$$\Leftrightarrow \begin{cases} B = 1 - A \\ -1 = \sqrt{2}A - \sqrt{2} + \sqrt{2}A \end{cases}$$

$$\Leftrightarrow \begin{cases} B = 1 - A \\ -1 + \sqrt{2} = 2\sqrt{2}A \end{cases}$$

$$\Leftrightarrow A = \frac{-1 + \sqrt{2}}{2\sqrt{2}} = \frac{-1}{2\sqrt{2}} + \frac{1}{2} = \frac{-2 + 2\sqrt{2}}{4\sqrt{2}}$$

$$\Leftrightarrow B = 1 + \frac{1}{2\sqrt{2}} - \frac{1}{2} = \frac{1}{2} + \frac{1}{2\sqrt{2}}$$

" $\sqrt{2}/4$

$$= \frac{x-1}{2} + \frac{-\sqrt{2}/2 + 1}{2(x-\sqrt{2})} + \frac{1 + \sqrt{2}/2}{2(x+\sqrt{2})}$$

$$= \frac{x-1}{2} - \frac{\sqrt{2}/2 - 1}{2(x-\sqrt{2})} + \frac{1 + \sqrt{2}/2}{2(x+\sqrt{2})}$$

oef. 2.17 (c)

$$c) \quad y = \frac{x}{1-x} \quad \leadsto \quad x = \frac{y}{1-y} \quad \text{voor } y \neq -1$$

oef. 2.22 (a, b)

$$a) \quad \begin{cases} x+y=3 \\ x-y=1 \end{cases} \Leftrightarrow \begin{cases} x=3-y \\ (3-y)-y=1 \end{cases} \Leftrightarrow \begin{cases} x=3-y \\ -2y=-2 \end{cases} \Leftrightarrow \begin{cases} x=2 \\ y=1 \end{cases}$$

$$c) \quad \begin{cases} 2x-y=2 \\ x^2-xy+y^2=1 \end{cases} \Leftrightarrow \begin{cases} 2x-2=y \\ x^2-x(2x-2)+(2x-2)^2=1 \end{cases}$$

$$\Leftrightarrow \begin{cases} 2x-2=y \\ x^2-2x^2+2x+4x^2-8x+4=1 \end{cases}$$

$$\Leftrightarrow \begin{cases} 2x-2=y \\ 3x^2-6x=-3 \end{cases} \Leftrightarrow \begin{cases} 2x-2=y \\ x^2-2x=-1 \end{cases}$$

$$b) \quad \begin{cases} 3x-2y=1 \\ 6x-4y=3 \end{cases} \Leftrightarrow \begin{cases} 6x-4y=2 \\ 6x=3+4y \end{cases} \Leftrightarrow \begin{cases} 3+4y-4y=2 \\ 6x=3+4y \end{cases}$$

$$\Leftrightarrow \begin{cases} 3=2 \\ 6x=3+4y \end{cases} \Rightarrow \text{VALS STELSEL}$$

oef. 3.1

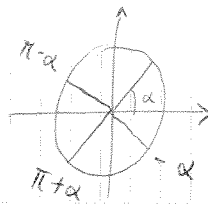
$$180^\circ = \pi$$

$$a) \quad A = 5^\circ = \frac{\pi}{36}$$

$$c) \quad C = 120^\circ = \frac{2\pi}{3}$$

$$b) \quad B = 87^\circ = \frac{87\pi}{180}$$

$$d) \quad D = 260^\circ = \frac{13\pi}{9}$$



oef. 3.7 (a, c)

$$a) \cos\left(x + \frac{\pi}{2}\right) \stackrel{AC}{=} -\sin x$$

$$c) \sin\left(x - \frac{\pi}{2}\right) \stackrel{C}{=} -\cos x$$

oef. 3.8 (a, c, j)

$$a) \sin 135^\circ = \sin \frac{135\pi}{180} = \sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$$

$$c) \tan\left(-\frac{5\pi}{4}\right) = -1$$

$$j) \arcsin \frac{\sqrt{2}}{2} = \frac{\pi}{4}$$

oef. 3.14 (a, c)

$$a) r = 3; \theta = \pi/3$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$x = 3 \cos \pi/3 = 3/2$$

$$y = 3 \sin \pi/3 = 3\sqrt{3}/2$$

$$c) r = 1; \theta = \pi$$

$$x = 1 \cdot \cos \pi = -1$$

$$y = 1 \cdot \sin \pi = 0$$

oef. 3.11

grafiek $\phi(x, t) = \sin(2\pi(x - t))$ met $-1 \leq x \leq 2$ voor $t = 0$
 $t = 1/4$
 $t = 1/2$

9/10/2019

oefening 3

oef. 2.17(e)

$$e) \quad y = \frac{x^2 - 1}{x^2 + 1}$$

~~Waarom?~~ ~~Waarom?~~ ~~Waarom?~~

$$y(x^2 + 1) = x^2 - 1$$

$$yx^2 + y = x^2 - 1$$

$$(y - 1)x^2 + y + 1 = 0$$

$$x^2 = \frac{-y - 1}{y - 1} = \frac{y + 1}{1 - y}$$

$$x = \pm \sqrt{\frac{y + 1}{1 - y}} \quad \text{met } -1 \leq y < 1$$

oef. 2.18

$$K_a = \frac{\alpha^2 C}{1 - \alpha}$$

$$K_a(1 - \alpha) = \alpha^2 C$$

$$K_a - K_a \alpha = \alpha^2 C$$

tweede grad. vgl

$$0 = \alpha^2 C + K_a \alpha - K_a$$

$$\rightarrow D = K_a^2 + 4CK_a$$

$$\rightarrow \alpha_{1,2} = \frac{-K_a \pm \sqrt{K_a^2 + 4CK_a}}{2C}$$

Def. 2.24 (b)

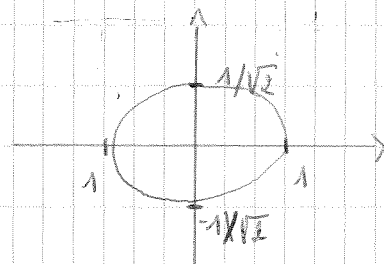
$$b) \begin{cases} x - 2xy + xy^2 = 0 \\ y + xy = 0 \end{cases} \Leftrightarrow \begin{cases} x + 2y + y^2 = 0 \\ xy = -y \end{cases}$$

$$\Leftrightarrow \begin{cases} x - 2xy + xy^2 = 0 \\ xy = -y \end{cases} \Leftrightarrow \begin{cases} -1 + 2y - y^2 = 0 \\ x = \frac{-y}{y} = -1 \end{cases}$$

$$\Leftrightarrow \begin{cases} y \Rightarrow D = 4 - 4 \cdot 1 \cdot 1 = 0 \\ y = \frac{-2}{-2} = 1 \\ x = -1 \end{cases}$$

Def. 2.21 (b)

niveauekrommen p. 38
 $\hookrightarrow$ omvormen naar $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



$$b) F(x, y) = x^2 + 2y^2 = 1$$
$$\Rightarrow a = 1$$
$$b = \sqrt{\frac{1}{2}}$$

Def. 3.9 (c)

$$c) \sin x = -\frac{1}{2}$$

$$x = \frac{-\pi}{6} + 2k\pi \quad \text{en} \quad x = \frac{7\pi}{6} + 2k\pi \quad \text{met } k \in \mathbb{Z}$$

Def. 3.15 (a, c)

$$r = \sqrt{x^2 + y^2}$$

$$\tan \theta = \frac{y}{x}$$

a) (3, 2)

$$\hookrightarrow r = \sqrt{9+4} = \sqrt{13}$$

$$\tan \theta = \frac{2}{3} \Leftrightarrow \theta = \tan^{-1} \frac{2}{3} =$$

c) (3, -2)

$$\hookrightarrow r = \sqrt{13}$$

$$\tan \theta = \frac{-2}{3} \Leftrightarrow \theta =$$

Def. 3.16 (a, b, e) → EXAMEN

~~max/min~~

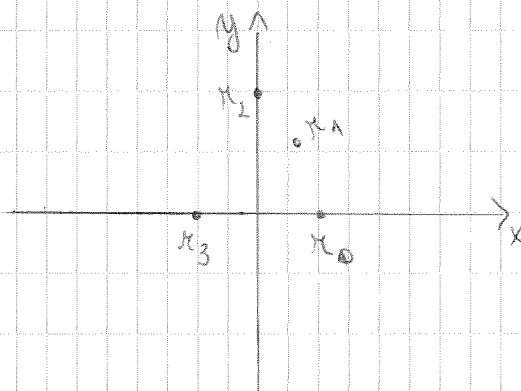
a) $r = 1 + \sin \theta$

$$\hookrightarrow (r_0, 0) = (1, 0)$$

$$(r_1, \frac{\pi}{4}) = (1 + \frac{\sqrt{2}}{2}, \frac{\pi}{4})$$

$$(r_2, \frac{\pi}{2}) = (2, \frac{\pi}{2})$$

$$(r_3, \pi) = (1, \pi)$$



e) $r = \frac{1}{\sin \theta}$

$$\hookrightarrow x = r \cos \theta$$

$$y = r \sin \theta$$

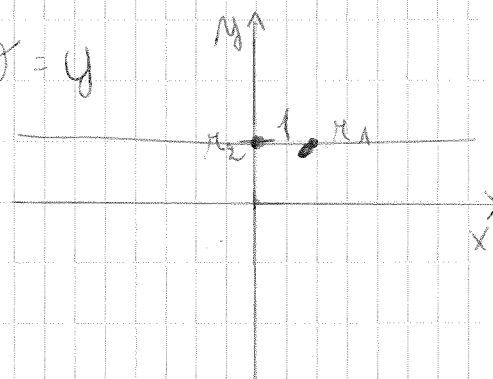
$$1 = r \sin \theta = y$$

$$\hookrightarrow (r_0, 0) = \text{X}$$

$$(r_1, \frac{\pi}{4}) = (\frac{2}{\frac{\sqrt{2}}{2}}, \frac{\pi}{4}) = (\sqrt{2}, \frac{\pi}{4})$$

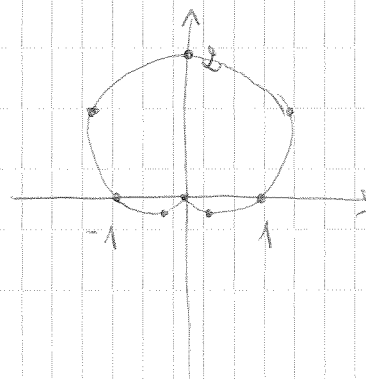
$$(r_2, \frac{\pi}{2}) = (1, \frac{\pi}{2})$$

$$(r_3, \pi) = \text{X}$$



θ	r
0	$1 + \sin(0) = 1$
$\pi/4$	$1 + \frac{\sqrt{2}}{2}$
$\pi/2$	2
$\vdots$	$\vdots$
$\frac{7\pi}{4}$	$1 - \frac{\sqrt{2}}{2}$

⇒



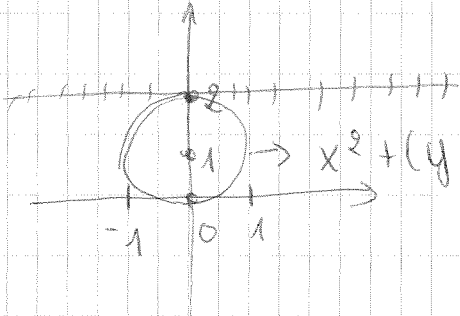
✓ Def. 3.17(b, e)

(b) $r = 2 \sin \theta$

$\hookrightarrow (r_0, \theta) = (0, 0) \leadsto \sin 0 \cdot 0 = y_0 = 0 \leadsto x_0 = 0$
 $(r_1, \frac{\pi}{4}) = (\sqrt{2}, \frac{\pi}{4}) \leadsto \sin \frac{\pi}{4} \cdot \sqrt{2} = y_1 = 1 \leadsto x_1 = 1$
 $(r_2, \frac{\pi}{2}) = (2, \frac{\pi}{2}) \leadsto \sin \frac{\pi}{2} \cdot 2 = y_2 = 2 \leadsto x_2 = 0$
 $(r_3, \pi) = (0, \pi) \leadsto \sin \pi \cdot 0 = y_3 = 0 \leadsto x_3 = -1$

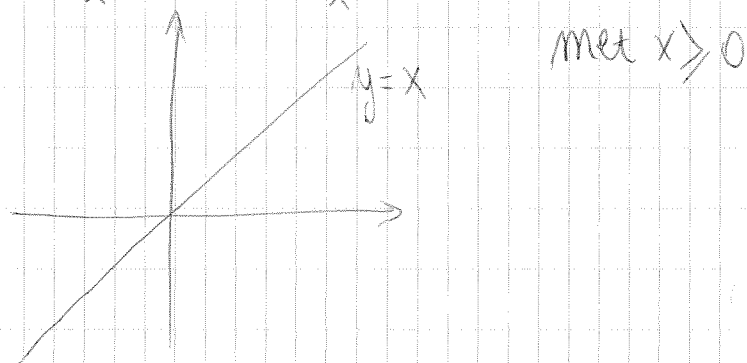
$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$

$\hookrightarrow \frac{r}{y} = \frac{2 \sin \theta}{\sin \theta} = 2$


 $x^2 + (y-1)^2 = 1$

e) $\theta = \frac{\pi}{4}$

$\Rightarrow \tan \theta = \frac{y}{x} \Leftrightarrow \tan \frac{\pi}{4} = \frac{y}{x} = 1 = \frac{y}{x} \Leftrightarrow y = x$



Def. 3.21 (a, c, i, l)

a) $\log_2 16 = 4$

c) $\log_2 \frac{1}{4} = -2$

i) $\ln e^{x^2} = x^2$

[illegible]

Def. 3.22 (a, d)

a) $\ln 2 + \ln 3 = \ln 6$

$$d) \ln(x+1) - 2 \ln x = \ln\left(\frac{x+1}{x^2}\right).$$

Def. 3.25 (a, b)

a) $\cosh x + \sinh x = e^x$, $\cosh x - \sinh x = e^{-x}$

$$\star \cosh x = \frac{e^x + e^{-x}}{2} \quad \text{or} \quad \sinh x = \frac{e^x - e^{-x}}{2}$$

$$\textcircled{a} \quad \frac{e^x + e^{-x}}{2} + \frac{e^x - e^{-x}}{2} = \frac{2e^x}{2} = e^x$$

$$(2) \rightarrow \frac{\cancel{e^x} + e^{-x}}{2} = \frac{\cancel{e^x} + e^{-x}}{2} = \frac{2e^{-x}}{2} = e^{-x}$$

b) $\cosh^2 x - \sinh^2 x = 1$

$$\left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 = \frac{\cancel{e^{2x}} + e^{-2x} - \cancel{e^{2x}} + e^{-2x}}{4} = \frac{2e^{-2x}}{4} = \frac{e^{-2x}}{2}$$

oef. 3.38 (c)

$$p(z) = az^2 + bz + c \quad \text{met } a \neq 0$$
$$z = x + yi$$

b) $z^2 + 2z + 5 = 0$

$$z_{1,2} = \frac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot 5}}{2}$$

$$\Rightarrow z_{1,2} = -1 + 2i \text{ en } -1 - 2i$$

$$\Delta \Rightarrow z_{1,2} = \frac{-b \pm w_1}{2a}$$
$$w^2 = 4 - 20 = -16 = -1 \cdot 16 = i^2 \cdot 16$$
$$\Leftrightarrow w_1 = \sqrt{i^2 \cdot 16} \text{ en } w_2 = -\sqrt{i^2 \cdot 16}$$

oef. 3.42 (a)

$$\rightarrow x + iy$$

a) $e^{i\pi/4}$

$$z = \frac{i\pi}{4}$$

$$e^z = e^{x+iy} = e^x (\cos y + i \sin y)$$

oef. 3.46

a) $e^z = 0$

$\hookrightarrow$ geen ope

$$|e^z| = e^x$$

b) $e^{iz} = 1$

$$e^{iy} = \cos y + i \sin y$$

11/10/2019 Refenztting 4 - 11/10/2019

Def. 3.23

$$a(t) = a(0) \cdot e^{-kt}$$

$$a(t_{1/2}) = \frac{1}{2}a(0) \quad (\text{einsetzen in } a(t))$$

$$a(t_{1/2}) = \frac{\ln 2}{k}$$

Def. 3.24(b)

$$b) \ln x = 2(\ln 3 - \ln 5)$$

$$\ln x = \ln 9 - \ln 25$$

$$\ln x = \ln\left(\frac{9}{25}\right) \Rightarrow x = \frac{9}{25}$$

Def. 3.33

$$f(t) = \frac{B}{1 + Ce^{-kt}}$$

$$a) * f(0) = \frac{1}{5} \frac{B}{1 + Ce^{-k0}} = \frac{1}{5} \frac{B}{5 + 5Ce^{-k0}} = \frac{B}{5 + 5C \cdot 1} = \frac{B}{5(1+C)}$$

$$* f(4) = \frac{1}{2} \frac{B}{1 + Ce^{-k4}} = \frac{1}{2} \frac{B}{1 + Ce^{-4k}}$$

✓

0

oef. 3.34

$$12\bar{z} + 4(z - \bar{z}) = 5 + 16i$$

$$z = x + iy$$

$$\Leftrightarrow 12\bar{z} + 4z - 4\bar{z} = 5 + 16i$$

$$\Leftrightarrow 12(\bar{z} + 4 - 4\bar{z}) = 5 + 16i$$

$$(x + iy)((x - iy) + 4 - 4(x - iy))$$

$$= (x + iy)(x - iy + 4 - 4x + 4iy)$$

$$= (x + iy)(-3x + 3iy + 4)$$

$$= -3x^2 + 3ixy + 4x - 3ixy + 3i^2y^2 + 4iy$$

$$= -3x^2 + 4x + 4iy - 3y^2$$

$$= x(-3x + 4) + iy(4 - 3y)$$

$$\Leftrightarrow (x + iy)(x - iy) + 4((x + iy) - (x - iy)) = 5 + 16i$$

$$\Leftrightarrow x^2 - i^2y^2 + 4(x + iy - x + iy) = 5 + 16i$$

$$\Leftrightarrow x^2 - i^2y^2 + 4(2iy) = 5 + 16i$$

$$\Leftrightarrow x^2 + y^2 + 8iy = 5 + 16i$$

(als 2 complexe getallen gelijk zijn aan elkaar, dan moet hun reël deel & imag. deel ook gelijk zijn aan elk.)

$$\Leftrightarrow \begin{cases} x^2 + y^2 = 5 \\ 8iy = 16i \end{cases} \Leftrightarrow \begin{cases} x^2 = 5 - 4 = 1 \\ y = 2 \end{cases} \Leftrightarrow x = 1 \text{ of } -1$$

$$\text{Dan } z_1 = -1 + 2i \text{ of } z_2 = 1 + 2i$$

✓

0

oef. 3.38 (d, 1)

$$a) z^2 = 1 + i \quad (\text{2 oplossingen})$$

$$\Leftrightarrow (x + iy)^2 = 1 + i$$

$$\Leftrightarrow x^2 + 2ixy + i^2y^2 = 1 + i$$

$$\Leftrightarrow x^2 + 2ixy - y^2 = 1 + i$$

$$\Leftrightarrow \begin{cases} x^2 - y^2 + 2ixy = 1 + i \\ x^2 - y^2 = 1 \\ 2xy = 1 \end{cases} \Leftrightarrow \begin{cases} x^2 - \frac{1}{4x^2} = 1 \\ y = \frac{1}{2x} \end{cases}$$

$$\Leftrightarrow \frac{4x^4 - 1}{4x^2} = 1 \Leftrightarrow 4x^4 - 1 = 4x^2 \Leftrightarrow 4x^4 - 4x^2 - 1 = 0$$

$$\text{Dan } z_1 = \frac{1}{2} + \frac{\sqrt{2}}{2} + i \cdot \frac{1}{1 + \sqrt{2}} \text{ en } z_2 = \frac{1}{2} - \frac{\sqrt{2}}{2} - i \cdot \frac{1}{1 + \sqrt{2}}$$

$$\begin{cases} z = x + iy \\ z = |z| \cdot e^{i\theta} \end{cases}$$

$$\begin{cases} \frac{1}{2} + \frac{\sqrt{2}}{2} \leadsto y = \frac{1}{1 + \sqrt{2}} \\ \frac{1}{2} - \frac{\sqrt{2}}{2} \leadsto y = \frac{1}{1 - \sqrt{2}} \end{cases}$$

vervolg
opz. 4

$$\frac{x}{r} = \cos \theta$$

$$\Leftrightarrow \theta = \cos^{-1} \left(\frac{x}{r} \right)$$

vervolg op 338 (d)

→ om z_1 en z_2 om te zetten naar een schrijfwijze, hebben we de modulus en het argument nodig

$$|1+i| = \sqrt{2} \Rightarrow |z| = r = \sqrt{x^2 + y^2}$$

$$\text{Arg}(1+i) = \frac{\pi}{4} (\theta) \Rightarrow r^2 e^{i2\theta} = \sqrt{2} e^{i\frac{\pi}{4}}$$

$$r = r e^{i\theta} \quad \cos^{-1} \left(\frac{\sqrt{2}}{2} \right) = \theta$$

$$\Rightarrow \begin{cases} r^2 = \sqrt{2} \\ 2\theta = \frac{\pi}{4} + 2k\pi \quad (k \in \mathbb{Z}) \end{cases} \Rightarrow \begin{cases} r = \sqrt[4]{2} \\ \theta = \frac{\pi}{8} + k\pi \quad (k \in \mathbb{Z}) \end{cases}$$

f) $z^4 = 1$ (4de'len)

$1, i, -1, -i$

$$|z| = \sqrt{1+0} = 1 = r$$

$$\text{Arg}(1) \Rightarrow \frac{1}{1} = \cos \theta \Leftrightarrow \theta = 0$$

$$z = r e^{i\theta}$$

$$\Rightarrow r^4 e^{i4\theta} = 1 \cdot e^{i0}$$

$$\Rightarrow \begin{cases} r^4 = 1 \\ 4\theta = 0 + 2k\pi \quad (k \in \mathbb{Z}) \end{cases}$$

$$\Leftrightarrow \begin{cases} r = 1 \\ \theta = 0 + \frac{1}{2} k\pi \quad (k \in \mathbb{Z}) \end{cases}$$

$$\begin{aligned} 1 \cdot e^{i0} &= 1 \\ 1 \cdot e^{i2\pi} &= 1 \cdot \left(\frac{e^{i\pi}}{e^{i\pi}} \right)^2 = 1 \\ 1 \cdot e^{i4\pi} &= 1 \\ 1 \cdot e^{i6\pi} &= 1 \end{aligned}$$

$$(z^n = r^n (\cos(n\theta) + i \sin(n\theta)))$$

$$z = r (\cos \theta + i \sin \theta)$$

$$z^2 = r^2 (\cos(2\theta) + i \sin(2\theta))$$

oef. 4.2 (a, c)

a) $y = 2x^2$

P: (1, 2)

✓ hiezo raaklijn = $y' = 4x$

✗ $y - 2 = 4(x - 1)$

$y = 4x - 2$

c) $y = x^2 + 2x$

P: (a, a^2 + 2a)

✓ hiezo raaklijn = $y' = 2x + 2$

✗ $y - (a^2 + 2a) = (2a + 2)(x - a)$

$y - a^2 - 2a = 2ax - 2a^2 + 2x - 2a$

$y = 2x(a + 1) - a^2$

oef. 4.4 (a, b)

a) $\lim_{x \rightarrow \infty} \frac{x+1}{x+3} = \lim_{x \rightarrow \infty} \frac{x(1 + \frac{1}{x})}{x(1 + \frac{3}{x})} = 1$

b) $\lim_{x \rightarrow \infty} \frac{3x^3 + 5x - 7}{2x^3 + 1} = \lim_{x \rightarrow \infty} \frac{x^3(3 + \frac{5}{x^2} - \frac{7}{x^3})}{x^3(2 + \frac{1}{x^3})} = \frac{3}{2}$

oef. 4.6 (f)

f) $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} \stackrel{H}{=} \lim_{x \rightarrow 1} \frac{3x^2}{1} = 3$

(*) Regel v.l'Hôpital $\Rightarrow \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$

ex. 4.7(a)

$$a) \lim_{x \rightarrow 0} \frac{\sin 3x}{x} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{3 \cos 3x}{1} = 3$$

ex. 4.9(b, d, f)

$$b) \left(\frac{3-x}{x} \right)' = \frac{-1x - (3-x) \cdot 1}{x^2} = \frac{-\cancel{x} - 3 + \cancel{x}}{x^2} = -\frac{3}{x^2}$$

$$d) (x \ln x - x)' = \left(\ln x + x \cdot \frac{1}{x} \right) - 1 = \ln x$$

$$f) \left(\sqrt[3]{x} \right)' = \left(x^{1/3} \right)' = \frac{1}{3} \cdot x^{-2/3} = \frac{1}{3 \sqrt[3]{x^2}}$$

ex. 4.10(c)

$$c) x^2 \sin x = 2x \sin x + x^2 \cos x$$

6/10/2019 Rechenhilfings (Wiskunde)

af. 4.4 (i, j)

$$\begin{aligned}
 i) \lim_{x \rightarrow +\infty} (\sqrt{x^2 + ax} - x) &= \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 + ax} - x}{1} \cdot \frac{\sqrt{x^2 + ax} + x}{\sqrt{x^2 + ax} + x} \\
 &= \lim_{x \rightarrow +\infty} \frac{(x^2 + ax) - x^2}{\sqrt{x^2 + ax} + x} = \lim_{x \rightarrow +\infty} \frac{ax}{\sqrt{x^2 + ax} + x} = \lim_{x \rightarrow +\infty} \frac{a}{\sqrt{1 + \frac{a}{x}} + 1} \\
 &= \frac{a}{2}
 \end{aligned}$$

$$\begin{aligned}
 j) \lim_{x \rightarrow -\infty} (\sqrt{x^2 + ax} - x) &= \lim_{x \rightarrow -\infty} -x \sqrt{1 + \frac{a}{x}} - x \quad (\text{wan } \sqrt{x^2} = |x| = \begin{cases} x & (x \geq 0) \\ -x & (x < 0) \end{cases}) \\
 &= +\infty
 \end{aligned}$$

af. 4.1 (a)

a) $y = \sqrt{x}$

$f(x) = y = \sqrt{x}$

$\Delta y = \sqrt{(x + \Delta x)} - \sqrt{x}$

$$\begin{aligned}
 \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} &= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x) - (x)}{\Delta x (\sqrt{x + \Delta x} + \sqrt{x})} = \lim_{\Delta x \rightarrow 0} \frac{\Delta x}{\Delta x (\sqrt{x + \Delta x} + \sqrt{x})} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}
 \end{aligned}$$

$$\begin{aligned}
 \Delta y &= f(x + \Delta x) - f(x) \\
 f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}
 \end{aligned}$$

def 4.7 (f)

tip: $\lim_{y \rightarrow 0} \frac{\sin y}{y} = 1$

$$f) \lim_{x \rightarrow \pi} \frac{\sin x}{x - \pi} = \lim_{x - \pi \rightarrow 0} \frac{\sin x}{x - \pi} \stackrel{AS}{=} \lim_{x - \pi \rightarrow 0} \frac{-\sin(x - \pi)}{x - \pi} = -1$$

def 4.9 (i, k, l)

$$i) (x^2 e^{2x^2+1})' = 2x e^{2x^2+1} + x^2 \cdot (4x e^{2x^2+1}) \\ = 2x e^{2x^2+1} + 4x^3 e^{2x^2+1}$$

$$k) \left(\sqrt{\frac{x-3}{x-2}} \right)' = \frac{1}{2} \left(\frac{x-3}{x-2} \right)^{-1/2} \left(\frac{x-3}{x-2} \right)' \\ = \frac{1}{2} \sqrt{\frac{x-2}{x-3}} \frac{x-2 - x+3}{(x-2)^2} = \frac{1}{2} \sqrt{\frac{x-2}{x-3}} \frac{1}{(x-2)^2} \\ = \frac{1}{2} \frac{1}{(x-3)^{1/2} (x-2)^{3/2}}$$

$$l) \left((x + \sqrt{1+x^2})^3 \right)' = 3 (x + \sqrt{1+x^2})^2 \left(1 + \frac{1}{\sqrt{1+x^2}} \cdot 2x \right)$$

def 4.10 (a)

$$a) \left(\frac{\sqrt{x} \ln x}{1+x^3} \right)' = \frac{\left(\frac{\ln x}{2\sqrt{x}} + \frac{\sqrt{x}}{x} \right) (1+x^3) - (\sqrt{x} \ln x) (3x^2)}{(1+x^3)^2}$$

4.12
4.33 (a, b)
4.34 (b)

oef. 4.13 (b, d, j)

$$b) (x^2 + 4x + 1)^6 = 6(x^2 + 4x + 1)^5 (2x + 4)$$

$$d) \ln(\sin 4x) = \frac{1}{\sin 4x} \cdot 4 \cos 4x = 4 \frac{\cos 4x}{\sin 4x}$$

$$j) \cos(\tan x) = -\sin(\tan x) \cdot \frac{1}{\cos^2 x}$$

oef. 4.14 (a)

$$a) \frac{d}{dx} \tan x = \frac{1}{\cos^2 x} = 1 + \tan^2 x$$

geg: $(\sin x)' = \cos x$
 $(\cos x)' = -\sin x$

quotiëntregel

$$\frac{d}{dx} \tan x = \frac{d}{dx} \left(\frac{\sin x}{\cos x} \right) = \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \xrightarrow{HF} = \frac{1}{\cos^2 x}$$

$$= \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = 1 + \tan^2 x \quad \square$$

oef. 4.24 (a)

$$a) f''(x) = ? \quad \text{met } f(x) = (x^3 + 1)^2$$

$$\hookrightarrow f'(x) = 2(x^3 + 1)(3x^2) = 6x^2(x^3 + 1) = 6x^5 + 6x^2$$

$$\hookrightarrow f''(x) = (f'(x))' = 30x^4 + 12x$$

oef. 4.27

$$y = A \cos x + B \sin x \leadsto \frac{d^2 y}{dx^2} = -y$$

$$*) \frac{dy}{dx} = -A \sin x + B \cos x$$

$$*) \frac{d^2 y}{dx^2} = \left(\frac{dy}{dx} \right)' = -A \cos x - B \sin x$$



oef. 4.30

→ structuur opschrijven op examen
+ conclusie niet vergeten

$$y = e^{-x^2}$$

a)

BS:

IS:

IH: voor een zekere waarde $p \in \mathbb{N}$ geldt

$$\frac{d^p y}{dx^p} = (-1)^p \cdot H_p(x) e^{-x^2}$$

TB: voor $p+1$ geldt dan

$$\frac{d^{p+1} y}{dx^{p+1}} = (-1)^{p+1} H_{p+1}(x) e^{-x^2} \quad (*)$$

$$\begin{aligned} \underline{B}: \quad L &= \frac{d}{dx} \left(\frac{d^p y}{dx^p} \right) \stackrel{IH}{=} \frac{d}{dx} \left((-1)^p H_p(x) e^{-x^2} \right) \\ &= (-1)^p H_p'(x) e^{-x^2} - (-1)^p H_p(x) \cdot 2x e^{-x^2} \\ &= (-1)^{p+1} (H_p(x) 2x - H_p'(x)) e^{-x^2} \end{aligned}$$

↳ dit is gelijk à RL van (*) als $H_p(x) 2x - H_p'(x)$
een veelterm is graad $p+1$ zou zijn
(wie is dat zo?)

conclusie



12/10/2019

Mathematik

Ableitungen

$$n \cdot x^{n-1} = -1 \cdot x^{-1-1} = -\frac{1}{x^2}$$

Def. 4.16 (b)

Tip: $\frac{df^{-1}}{dx} = \frac{1}{\frac{df}{dy}}$

$$b) \frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$f^{-1}(x) = \arccos x \quad | \quad f(y) = \cos y$$

$$\frac{d}{dx} \underbrace{\cos^{-1} x}_{\arccos(x)} = \frac{1}{\frac{d \cos y}{dy}} = \frac{1}{-\sin y} = \frac{-1}{\sqrt{1-\cos^2 y}}$$

$$\text{HF: } \sin^2 \left(\frac{x}{y} \right) + \cos^2 \left(\frac{x}{y} \right) = 1$$

$$= \frac{-1}{\sqrt{1-x^2}}$$

$$\text{wobei } y = f^{-1}(x) \\ (= \arccos x)$$

Def. 4.20

Tip: $f'(x_0) \approx \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$ als Δx klein

$$y = x + \frac{1}{x} \rightarrow y' = 1 - \frac{1}{x^2} \quad \Delta x = 0,1$$

$$\Delta y = f(2,1) - f(2) = 0,07619 \quad (\text{exact result})$$

$$\rightarrow \Delta y \approx \underbrace{f'(x)}_{f'(2)} \underbrace{\Delta x}_{0,1} \approx \left(1 - \frac{1}{4} \right) \cdot 0,1 = 0,075$$

Def. 4.24 (b,c)

(p. 105 = Theorem)

$$b) f(x) = x^{532} - 5x^{37} - 4 \rightarrow \text{1469. ste Ableitung}$$

↳ ist größer als 532

$$f^{(1469)} = 0$$

c) $g(\theta) = \cos(2\theta) \rightarrow$ 2012 nu afgeleide

$$g'(2012) = (-1)^{2012/2} \cdot 2^{2012} \cdot \cos 2\theta$$

$$= 2^{2012} \cos 2\theta$$

even $f^e \Rightarrow f(x) = f(-x)$
 oneven $\Rightarrow f(x) = -f(-x)$

oef. 4.28 (a)

tip: gebruik kettingregel: $\frac{d(f \circ g)(x)}{dx}$

$$= \frac{df(q(x))}{dx} \cdot \frac{dq(x)}{dx}$$

a) als f = even, dan f' = oneven

$$\hookrightarrow \underbrace{f(x)}_{\text{"x" f}} = \underbrace{f(-x)}_{\text{"f(q(x))"}}$$

$$\hookrightarrow \approx f'(x) = -f'(-x)$$

$$q(x) = -x$$

$$\Rightarrow \frac{d(f(q(x)))}{dx} \cdot \frac{dq(x)}{dx} = \frac{d}{dx} (f(-x)) \cdot \frac{d}{dx} q(x)$$

$$\frac{d(f \circ g)(x)}{dx}$$

in pos. door even macht
 ↓
 even $\rightarrow$ oneven
 $\frac{d}{dx} (-x)$

oef. 5.4 (b, c)

b) $f(x) = -x^2 + x$ $[-1, 2]$
 $f'(x) = -2x + 1 \Rightarrow f'(x) = 0$
 $\Leftrightarrow x = \frac{1}{2} \text{ (1v)}$

x	-1	$\frac{1}{2}$	2
$f'(x)$	+	0	-
$f(x)$	-2	$\frac{1}{4}$	-2

abs. max in $x = \frac{1}{2}$

abs. min in $x = -1$ en $x = 2$

stat. punt: $x = \frac{1}{2}$

c) $f(x) = (x-4)^4$ $[3, 6]$
 $f'(x) = 4(x-4)^3 \Rightarrow f'(x) = 0$
 $\Leftrightarrow x = 4 \text{ (3v)}$

x	3	4	6
$f'(x)$	-	0	+
$f(x)$	1	0	16

abs. min in $x = 4$

abs. max in $x = 6$

rel. max in $x = 3$

stat. punt: $x = 4$

oef. 5.6(a)

a) $f(x) = \frac{1}{3}x^3 - x + 2$

$f''(x) = 2x$

$\hookrightarrow f''(-1) = -2 (< 0)$

$f''(1) = 2 (> 0)$

$f'(x) = x^2 - 1 \Rightarrow f'(x) = 0$
 $\Leftrightarrow x = \pm 1 \text{ (1v)}$

x	-1	1
$f'(x)$	+	-
$f(x)$	$\frac{2}{3}$	$\frac{4}{3}$
	lok. max.	lok. min.

abs. max in $x = -1$

abs. min in $x = 1$

stat. punten: $x = \pm 1$

oef. 5.8(a)

$$U(x) = \frac{A}{x^{12}} - \frac{B}{x^6} \quad \text{met } x > 0 \text{ en } A \text{ en } B = \text{pos. etc}$$

a) $R_e \Rightarrow$ waarvoor $U(x)$ min is
 $D_e = -U(R_e)$

$$\begin{aligned} * U'(x) &= A(x^{-12})' - B(x^{-6})' \\ &= A(-12 \cdot x^{-11}) - B(-6x^{-5}) \\ &= -12A x^{-11} + 6B x^{-5} \\ &= (-2A x^{-6} + B) x^{-5} \end{aligned}$$

$$\rightarrow U'(x) = 0 \Leftrightarrow x \neq 0 \rightarrow (\text{want } x > 0) \\ x = \frac{2A}{B} \quad (6v)$$

x	$\frac{2A}{B}$
$U(x)$	— 0 —
$U(x)$	$\underbrace{\hspace{2cm}}_{\text{min?}}$

$$\leadsto R_e = \left(\frac{2A}{B} \right)^{-6}{}^{1/6}$$

$$* D_e = - \left(\frac{A}{\left(\frac{2A}{B} \right)} - \frac{B}{\left(\frac{2A}{B} \right)} \right)$$

oef. 5.9(b)

$$b) a \cdot b = \max = f(b)$$

$$\hookrightarrow a + b = 20 \Leftrightarrow a = 20 - b$$

$$* f(b) = (20 - b) \cdot b = -b^2 + 20b$$

$$* f'(b) = -2b + 20 \Rightarrow f'(b) = 0 \Leftrightarrow b = 10$$

b	10
$f'(b)$	$+ 0 -$
	max

$\leadsto b = 10$ (bij max) dus $a = 20 - 10 = 10$
dus het product is 100

23/10/2019

Wiskunde 1-oefeningen

oefenzitting 7

oef. 5.3(b)b) $\log x \leq x$ voor alle $x \geq 0$ met eig. 5.1.2 $\Rightarrow f(x) = \log x$

$$g(x) = x$$

$$\Leftrightarrow f(a) \leq g(a) \quad \text{bv } a = 1 \text{ en } \pi/4 \leq 1$$

$$\Leftrightarrow f'(x) \leq g'(x) \quad \text{met } x \in [a, b]$$

oef. 5.4(a, g)

a) $f(x) = \sqrt{1+x^2} \quad [0, \sqrt{8}]$

$$f'(x) = \frac{1}{2} \frac{2x}{\sqrt{1+x^2}} \Rightarrow f'(x) = 0$$

$$\Leftrightarrow x = 0$$

x	0		$\sqrt{8}$
$f'(x)$	0	+	+
$f(x)$	1	$\uparrow$	3

stat. punt $x = 0$ abs. min $x = 0$ abs. max $x = \sqrt{8}$

g) $f(x) = x \ln x \quad \text{met } x > 0$

$$f'(x) = \ln x + 1 \Rightarrow f'(x) = 0$$

$$\Leftrightarrow x = e^{-1}$$

$$\ln x = -1$$

stat. punt: $x = e^{-1}$ abs. min: $x = e^{-1}$

op max.

oef. 5.6(b)

b) $f(x) = x \ln x$

$$f'(x) = \ln x + 1$$

$$\Leftrightarrow f'(x) = 0$$

$$\Leftrightarrow x = e^{-1}$$

$$f''(x) = \frac{1}{x}$$

$\leadsto$ min in e^{-1}

oef. 5.7(a)

d) $q(x) = \sqrt{x^2+1} - ax$ met $-1 < a < 1$

$$\frac{x}{\sqrt{x^2+1}} - a = \frac{x - a\sqrt{x^2+1}}{\sqrt{x^2+1}}$$

$$\Leftrightarrow x - a\sqrt{x^2+1} = 0$$

$$q'(x) = \frac{1}{\sqrt{x^2+1}} - a \Rightarrow q'(x) = 0$$

$$\Leftrightarrow x = a\sqrt{x^2+1} = a^2(x^2+1) = a^2x^2 + a^2$$

$$\Leftrightarrow x = \pm \sqrt{\frac{a^2}{1-a^2}}$$

* $a=0$: $x=0 \leadsto$ globaal min in 0

* $-1 < a < 0$: omdat $x = a\sqrt{x^2+1}$ (zie berekening $q'(x)=0$),

is in dit geval $x = -\sqrt{\frac{a^2}{1-a^2}} = \frac{a}{\sqrt{1-a^2}} \Rightarrow \sqrt{a^2} = |a|$
 $\begin{matrix} a < 0 & -a \\ a > 0 & a \end{matrix}$

oef. 5.9(a)

a) $x^2 - y^2 = 1$

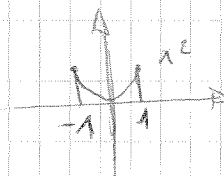
P: $(0, r)$

tip: - Denk aan form. voor de afstand

van punt (x, y) tot $(0, r)$

- gebruik vgl. hyperbool om 1 variabele te beschrijven

- een minimum van f is ook min van f^2



oef. 5.20 (b, c)

b) $\sqrt{1+x}$ $\rightarrow$ Taylorveelterm $P_n(x) = \sum_{k=0}^n \frac{f^{(k)}(0)}{k!} x^k$

$$= f(0) + f'(0)x + \frac{f''(0)}{2!} x^2 + \dots + \frac{f^{(n)}(0)}{n!} x^n$$

$$P_4(x) = f(0) + f'(0)x + \frac{f''(0)}{2!} x^2 + \frac{f^{(3)}(0)}{3!} x^3 + \frac{f^{(4)}(0)}{4!} x^4$$

$$= \sqrt{1} + \frac{1}{2} x + \frac{-1}{4} \cdot \frac{1}{2} x^2 + \frac{3}{8} \cdot \frac{1}{6} x^3 + \frac{-15}{16} \cdot \frac{1}{24} x^4$$

$$= \frac{-15}{384} = \frac{-5}{128}$$

$$* f'(x) = \frac{1}{2\sqrt{1+x}} \quad \leadsto f'(0) = \frac{1}{2}$$

$$* f''(x) = \frac{1}{2} \cdot \frac{-1}{2\sqrt{(1+x)^3}} \quad \leadsto f''(0) = \frac{-1}{4}$$

$$* f^{(3)}(x) = \frac{-1}{4} \cdot \frac{-3}{2\sqrt{(1+x)^5}} \quad \leadsto f^{(3)}(0) = \frac{3}{8}$$

$$* f^{(4)}(x) = \frac{3}{8} \cdot \frac{-5}{2\sqrt{(1+x)^7}} \quad \leadsto f^{(4)}(0) = \frac{-15}{16}$$

$$\Rightarrow P_4(x) = 1 + \frac{1}{2} x - \frac{1}{8} x^2 + \frac{1}{16} x^3 - \frac{5}{128} x^4$$

c) $\ln(1+x) = f(x) \leadsto f(0) = 0$

$$* f'(x) = \frac{1}{1+x} \quad \leadsto f'(0) = 1$$

$$* f''(x) = -\frac{1}{(1+x)^2} \quad \leadsto f''(0) = -1$$

$$* f^{(3)}(x) = \frac{2}{(1+x)^3} \quad \leadsto f^{(3)}(0) = 2$$

$$* f^{(4)}(x) = -\frac{6}{(1+x)^4} \quad \leadsto f^{(4)}(0) = -6$$

$$\Rightarrow P_4(x) = 0 + x + 0x^2 + \frac{-1}{2!} x^2 + \frac{2}{3!} x^3 + \frac{-6}{4!} x^4$$

$$= x - \frac{1}{2} x^2 + \frac{1}{3} x^3 - \frac{1}{2} x^4$$

Def. 5.23 (a)

a) $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} \rightsquigarrow$ TV von Zähler & TV v. Nenner

$\checkmark \underline{T}: 1 + 1 + 1 + 1 + \dots$

$\times \underline{N}: 1$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{1 + 1 + 1 + \dots}{1} = 1$$

Def. 5.24 (c, j)

c) $\lim_{x \rightarrow \infty} \frac{e^x}{x^2} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{e^x}{2x} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{e^x}{2} = +\infty$

j) $\lim_{x \rightarrow \infty} \frac{e^x}{x^3 + 2x^2 + 1} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{e^x}{3x^2 + 4x} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{e^x}{6x + 4} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{e^x}{6} = 0$

Def. 5.25 (a, b)

a) $\lim_{x \rightarrow \infty} \frac{4x - 3}{2x + 4} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{4}{2} = \frac{4}{2} = 2$

b) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} \stackrel{H}{=} \lim_{x \rightarrow 2} \frac{2x}{1} = 4$

25/10/2019

Wiskunde 1-oefeningenoefening 8oef. 5.17(c)

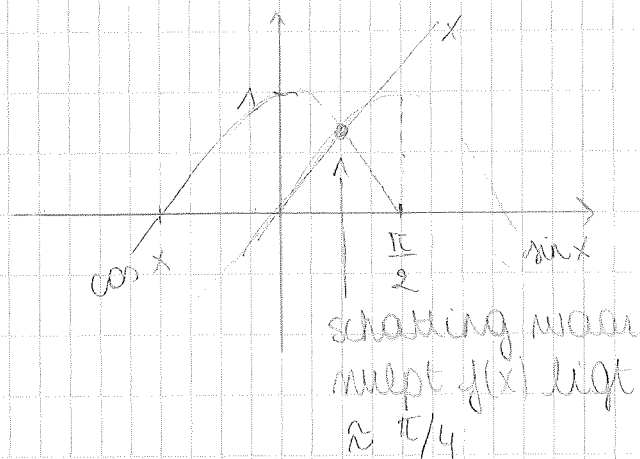
tip: (5.17 & 5.18): schets de grafiek om een geschikte startwaarde te vinden

$$c) f(x) = x \sin x - \cos x$$

$$x \in [0, \frac{\pi}{2}]$$

$$f'(x) = \sin x + x \cos x + \sin x$$

$$= 2 \sin x + x \cos x$$



$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$① \quad x_{k+1} = \frac{\pi}{4} - \frac{\frac{\pi}{4} \sin(\frac{\pi}{4}) - \cos(\frac{\pi}{4})}{2 \sin(\frac{\pi}{4}) + \frac{\pi}{4} \cos(\frac{\pi}{4})} = 0,8624434632$$

$\parallel$
 x_1

$$② \quad x_2 = 0,8603349794$$

$$③ \quad x_3 = 0,860333589$$

$\Rightarrow$ nulpunt in $x = 0,86033$

oef. 5.18(a)

$$a) f(x) = x^3 - x + 3$$

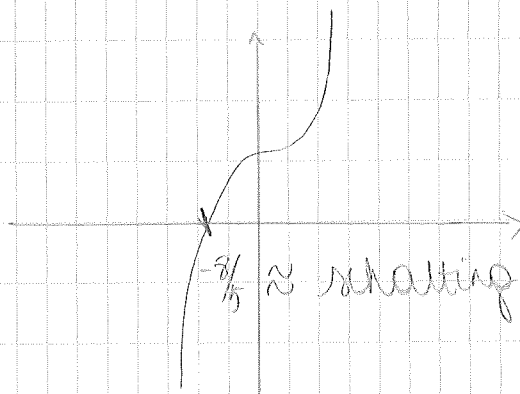
$$f'(x) = 3x^2 - 1$$

$$x_1 = \frac{-8}{5} - \frac{(-\frac{8}{5})^3 - (-\frac{8}{5}) + 3}{3(-\frac{8}{5})^2 - 1} = -1,675449102$$

$$x_2 = -1,671709395$$

$$x_3 = -1,671699882$$

$$x_4 = -1,671699882$$



$\Rightarrow$ nulpunt in $x = -1,6716$

oef. 5.20 (b, e)

tip: De Taylorveelterm van een functie f rond $x = a$ is geg. door

$$P_n(x) = \sum_{k=0}^n \frac{f^{(k)}(a)}{k!} (x-a)^k$$

$$= \frac{f(a)}{0!} + \frac{f'(a)}{1!} (x-a) + \dots + \frac{f^{(n)}(a)}{n!} (x-a)^n$$

(vind de uitdrukkingen v. $f^{(k)}(a)$ eerst)

b) + e) $\rightarrow$ zie voorverduiding

bij 5.20 is $a=0$,
maar als $a=1$ (bv.) dan
met $(x-1)^{2/3, 4, \dots}$ staan

oef. 5.23 (c, d)

tip: kijk naar nb 5.4.6 op p. 133:

voor de substitutie $y = 1+x$ uit
OF vind Taylor veelterm v. $\ln(x)$
rond 1 direct

c) $\lim_{x \rightarrow 1} \frac{\ln x}{x^2 - 1}$

* T: $0 + 1 - \frac{1}{1} - \dots$

* N: $0 + 2 + 2$
(2x)

} eerste Taylorveelterm

$$\Rightarrow \lim_{x \rightarrow 1} \frac{1 + \dots}{2 + \dots} = \frac{1}{2}$$

-1 d) $\lim_{x \rightarrow 0} \frac{\cosh x - 1}{\cos x - 1}$

$$(\cosh x)' = \sinh x$$

* T: $0 + (-1) + \dots$

* N: $0 - (-1) + \dots$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{-1 + \dots}{1 + \dots} = -1$$

$$f'(x) \rightarrow \sinh x$$

$$f'(x) \rightarrow -\sin x$$

Def. 5.25 (q)

tip: gebruik dat $f(x) = e^{\ln(f(x))}$

q) $\lim_{x \rightarrow \infty} x^{1/x}$

$$= \lim_{x \rightarrow \infty} e^{\ln(x^{1/x})} = \lim_{x \rightarrow \infty} e^{\frac{1}{x} \ln x} = 1$$

$\frac{1}{\infty} \ln \infty = 0 \Rightarrow e^0$

Def 5.27 (a)

a) $\lim_{n \rightarrow \infty} \left(\frac{n+1}{n} \right)^n = \lim_{n \rightarrow \infty} \frac{(n+1)^n}{n^n} \stackrel{H}{=} \lim_{n \rightarrow \infty} \frac{n(n+1)^{n-1}}{n \cdot n^{n-1}} = \lim_{n \rightarrow \infty} \frac{(n+1)^{n-1}}{n^{n-1}}$

$\stackrel{H}{=} \lim_{n \rightarrow \infty} \frac{1}{1} = 1$

$\lim_{n \rightarrow \infty} \left(\frac{n+1}{n} \right)^n = \lim_{n \rightarrow \infty} e^{n \ln \left(\frac{n+1}{n} \right)} = \lim_{n \rightarrow \infty} e^{n [\ln(n+1) - \ln(n)]}$

$\stackrel{H}{=} \lim_{n \rightarrow \infty} \frac{n \cdot \frac{1}{n+1}}{-\frac{1}{n^2}}$

$\lim_{n \rightarrow \infty} \left(\frac{\ln \frac{n+1}{n}}{1/n} \right)$

$\frac{n}{n+1} \cdot \frac{n^2}{-1} = \frac{n^3}{-n-1}$

$\stackrel{H}{=} \lim_{n \rightarrow \infty} \frac{3n^2}{-1}$

6.1 (a, f, g, h) / 6.2 (c, f) / 6.3 (a) / 6.4 (b) / 6.7

voorbereiding wiskunde-oefeningen

oefenzitting 9-6/11/2019

oef 6.1 (a, f, g, h)

$$d) \int \frac{dt}{bt-1} = \frac{1}{b} \int \frac{1}{u} du = \frac{1}{b} \ln |bt-1| + C$$

$$bt-1 = u$$

$$bdt = du$$

$$f) \int (5s^4 + 2t + 3) ds = s^5 + 2ts + 3s$$

$$g) \int (5s^4 + 2t + 3) dt = 5s^4 t + t^2 + 3t$$

$$h) \int \frac{3x^3 + 2x + 1}{x^2} dx = \int 3 dx + \int \frac{2}{x} dx + \int \frac{1}{x^2} dx$$

$$= 3x + \ln 2 - \frac{1}{x} + C$$

oef 6.2 (c, f)

$$c) \int_a^{2a} \frac{1}{t} dt = [\ln |t|]_a^{2a} = \ln 2a - \ln a = \ln 2$$

$$f) \int_0^{2\pi} \cos(m\theta) d\theta \quad \text{met } m \in \mathbb{N}$$

$$* \underline{m=0} : \int_0^{2\pi} \cos 0 d\theta = [\theta]_0^{2\pi} = 2\pi$$

$$* \underline{m \neq 0} : \int_0^{2\pi} \cos(m\theta) d\theta = \left[\frac{\sin(m\theta)}{m} \right]_0^{2\pi} = \frac{\sin(m \cdot 2\pi)}{m} - 0$$

$$= 0$$

oef 6.3 (a)

$$a) \int_0^3 [x] dx = 0 + 1 + 2 = 3$$

oef 6.4 (b)

b) $f(x) = x^3$ met $a=1$ en $b=5$

$$\text{opp} = \int_1^5 f(x) dx = \int_1^5 x^3 dx = \left[\frac{x^4}{4} \right]_1^5 = 156$$

oef 6.7 $\rightarrow$ functie even of oneven

$$f(x) = f(-x) \Rightarrow \text{even}$$
$$f(-x) = -f(x) \Rightarrow \text{oneven}$$

~~a) $f(x) = |x| \sim \int_{-a}^a f(x) dx = \int_{-a}^a |x| dx = \left[\frac{|x|^2}{2} \right]_{-a}^a$~~

~~$f(x) = f(-x)$~~

~~$\Rightarrow \text{even}$~~

~~$= \frac{|a|^2}{2} - \frac{|-a|^2}{2}$~~

b) $f(x) = \frac{x}{1+x^2} \sim \int_{-a}^a f(x) dx = \int_{-a}^a \frac{x}{1+x^2} dx = \frac{1}{2} \int_{-a}^a \frac{1}{t} dt = 0$

$\Rightarrow \text{oneven}$

~~$2x dx = dt$~~

~~$x = -a \rightarrow t = 1+a^2$~~

~~$x = a \rightarrow t = 1+a^2$~~

~~$\Rightarrow$ dezelfde grenzen~~

~~c) $f(x) = x \cos x \sim \int_{-a}^a x \cos x dx = \left[x \sin x \right]_{-a}^a - \int_{-a}^a \sin x dx$~~

~~$f(-x) = -x \cdot \cos(-x)$~~

~~$f(-x) = -f(x)$~~

~~$u(x) = x$~~

~~$w'(x) = \cos x \Rightarrow w(x) = \sin x$~~

~~$= \left[x \sin x \right]_{-a}^a - \left[\cos x \right]_{-a}^a$~~

~~$\Rightarrow \text{oneven}$~~

d) $f(x) = x \sin x$

$f(-x) = -x \cdot \sin(-x)$

$= -x \cdot (-\sin x)$

$= x \sin x$

$\Rightarrow \text{even}$

e) $f(x) = e^x$ $f(-x) = e^{-x} \rightarrow$ noch even, noch oneven

f) $f(x) = \ln \tan x$ $f(-x) = \ln \tan(-x)$

$= -\ln \tan x$

$\Rightarrow \text{oneven}$

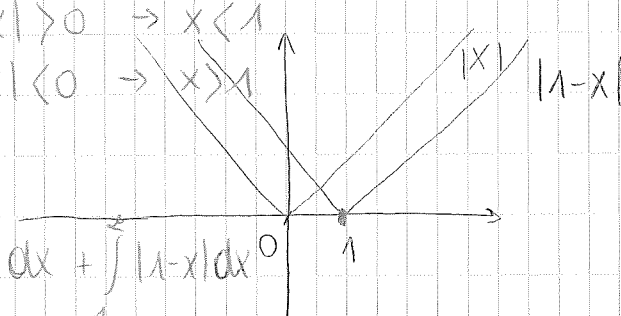
6/11/2019

Wiskunde 1-oefeningen

oefenzitting 9

oef. 6.2 (e)

Tip: Maak een schets en let op de verschillende uitdrukkingen v.d. absolute waarden in verschillende intervallen van x

$$\begin{aligned}
 \text{e)} \quad & \int_{-1}^2 (|x| + |1-x|) dx \\
 &= \int_{-1}^0 |x| dx + \int_{-1}^2 |1-x| dx \\
 &= \int_{-1}^0 |x| dx + \int_0^2 |x| dx + \int_{-1}^1 |1-x| dx + \int_1^2 |1-x| dx \\
 &= -\int_{-1}^0 x dx + \int_0^2 x dx + \int_{-1}^1 (1-x) dx - \int_1^2 (1-x) dx \\
 &= -\left[\frac{x^2}{2}\right]_{-1}^0 + \left[\frac{x^2}{2}\right]_0^2 + \left[x - \frac{x^2}{2}\right]_{-1}^1 - \left[x - \frac{x^2}{2}\right]_1^2 \\
 &= +\frac{1}{2} + 2 + \frac{1}{2} + \frac{3}{2} + \frac{1}{2} = 4
 \end{aligned}$$


oef. 6.3 (b)

Tip: $[x]$ is de grootste $n \in \mathbb{Z}$ waarvoor $n \leq x$. Maak een schets.

$$\begin{aligned}
 \text{b)} \quad & \int_0^3 [x^2] dx \sim \int_0^3 [x^2] dx \\
 & \hookrightarrow \text{als er is geïntegreerd tot 0 en 1 is } x=0; \text{ tot 1 en 2} \rightarrow x=1 \\
 & \rightarrow \text{maar het is } x^2 \text{ dus } \sqrt{\cdot} \text{ pakken} \\
 & (\text{bv. } [x] \text{ voor } 3,2 = 3) \\
 & \hookrightarrow \text{grootste geheel getal } (3,2)
 \end{aligned}$$

$$\begin{aligned}
 &= \int_0^{\sqrt{1}} 0 dx + \int_{\sqrt{1}}^{\sqrt{2}} 1 dx + \int_{\sqrt{2}}^{\sqrt{3}} 2 dx + \int_{\sqrt{3}}^{\sqrt{4}} 3 dx + \int_{\sqrt{4}}^{\sqrt{5}} 4 dx + \int_{\sqrt{5}}^{\sqrt{6}} 5 dx \\
 &+ \int_{\sqrt{6}}^{\sqrt{7}} 6 dx + \int_{\sqrt{7}}^{\sqrt{8}} 7 dx + \int_{\sqrt{8}}^3 8 dx = \dots
 \end{aligned}$$

$$= 7,694$$

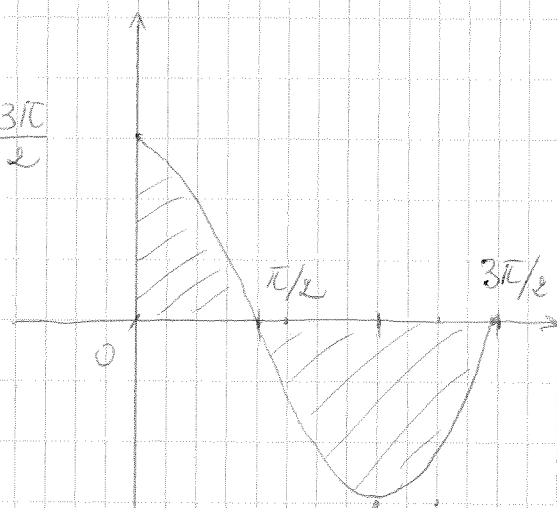
oef. 6.4 (a)

a) $f(x) = \cos x$ met $a = 0$ en $b = \frac{3\pi}{2}$

$$\text{opp} = \int_0^{\pi/2} (\cos x) dx - \int_{\pi/2}^{3\pi/2} (\cos x) dx$$

$$= [\sin x]_0^{\pi/2} - [\sin x]_{\pi/2}^{3\pi/2}$$

$$= 1 - 0 - [(-1) - 1] = 1 + 1 + 1 = 3$$

oef 6.5 (b, c)

Tip: zoek de snijpunten $\times$ maak daar een schets

b) $f_1(x) = 5x$ en $f_2(x) = x^3 - 4x$

$\times$ snijpunten: $5x = x^3 - 4x$
 $0 = x^3 - 9x$
 $0 = x(x^2 - 9)$
 $0 = x(x-3)(x+3)$
 $\Rightarrow x=0; x=-3; x=3$

$\times$ opp = $2 \cdot \int_0^3 (f_2(x) - f_1(x)) dx$ $5x - x^3 + 4x$

$$= 2 \cdot \int_0^3 (x^3 - 9x) dx = 2 \cdot \left[\frac{-x^4}{4} + \frac{9x^2}{2} \right]_0^3$$

$$= 2 \cdot \left(\frac{-81}{4} + \frac{9 \cdot 9}{2} \right) = +40,5 = \frac{+81}{2}$$

c) $f_1(x) = |x|$ en $f_2(x) = x^2 - 2$

$\times$ snijpunten: $|x| = x^2 - 2$

$x > 0 \Rightarrow x = x^2 - 2 \Leftrightarrow 0 = x^2 - x - 2$
 $x = 2$ of $x = -1$
 $x < 0 \Rightarrow -x = x^2 - 2 \Leftrightarrow 0 = x^2 + x - 2$
 $x = 1$ of $x = -2$

$\times$ opp = $-\int_{-2}^0 (x^2 + x - 2) dx + \int_0^2 (x^2 - x + 2) dx$

$$= -\left[\frac{x^3}{3} + \frac{x^2}{2} - 2x \right]_{-2}^0 + \left[\frac{x^3}{3} - \frac{x^2}{2} + 2x \right]_0^2 = \frac{20}{3}$$

Oef. 6.8

Tip: Kijk naar Eig. 6.3.2 op p. 158

$$\int_{-2}^2 f(x) dx = 0 \quad ?$$

→ dit geldt alleen voor oneven functies

Oef. 6.9 (a, e)

Tip Kijk naar Stellingen 6.4.1 op p. 160
en 6.4.4 op p. 163

a) $f(x) = \int_0^x \frac{\sin 2t}{1+t^2} dt$

voorbereiding wiskunde - oefeningen

oefenzitting 10 - 8/11/2019

oef. 6.16 (a, e)

x a) $\int_0^{\infty} e^{-3t} dt \Rightarrow$ voor elke $b > 0$ geldt: $\int_0^b e^{-3t} dt = \left[-\frac{1}{3} e^{-3t} \right]_0^b$
 $= -\frac{e^{-3b}}{3} + \frac{1}{3}$

als $b \rightarrow \infty$, dan $e^{-3b} \rightarrow 0$ en dus
 $\int_0^{\infty} e^{-3t} dt = -\frac{1}{3}$

e) $\int_0^1 x^{-1/3} dx = \left[\frac{x^{2/3}}{2/3} \right]_0^1 = \frac{3}{2}$

oef. 6.17

a) $\frac{d}{dx} (x \ln x - x) = \ln x$

$\Rightarrow \int \ln x \stackrel{!}{=} x \ln x - x$

$= \int 1 \cdot \ln x = x \ln x - \int \frac{1}{x} \cdot x dx = x \ln x - x (+C)$

b) $\int_0^1 \ln x dx = [x \ln x - x]_0^1 = -1$

8/11/2019

Wiskunde I - oefeningen

oefenzitting 10

oef. 6.11 (b)

$$b) \int_0^x \ln(1+t) dt = f(x)$$

$$\frac{d}{dx} \int_{a(x)}^{b(x)} f(t) dt = f(b(x)) \frac{db}{dx} - f(a(x)) \frac{da}{dx}$$

$$\leadsto f(0) = 0$$

$$\star \frac{d}{dx} \int_0^x \ln(1+t) dt = 1 \cdot \ln(1+x) - 0 \leadsto f'(0) = 0$$

$$\star \frac{d^2}{dx^2} \int_0^x \ln(1+t) dt = \frac{1}{1+x} \leadsto f''(0) = 1$$

$$\star \frac{d^3}{dx^3} \int_0^x \ln(1+t) dt = \frac{-1}{(1+x)^2} \leadsto f^{(3)}(0) = -1$$

$$\star \frac{d^4}{dx^4} \int_0^x \ln(1+t) dt = \frac{1 \cdot 2(1+x)}{(1+x)^4} = \frac{2+2x}{(1+x)^4} \leadsto f^{(4)}(0) = 2$$

$$\Rightarrow \int_0^x \ln(1+t) dt = \frac{x^2}{2} - \frac{x^3}{6} + \frac{2x^4}{24} = \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{12}$$

oef. 6.14 (b)

$$b) \lim_{x \rightarrow \infty} \int_0^{2x} \sqrt{1+t^2} dt$$

Tip: $\sqrt{1+t^2} > \sqrt{t^2}$ Dus $\int \sqrt{1+t^2} dt > \int \sqrt{t^2} dt$
 Oneigenlijke integraal

$$\int_a^{\infty} f(x) dx = \lim_{b \rightarrow \infty} \int_a^b f(x) dx$$

$$\int_0^{2x} \sqrt{1+t^2} dt \rightarrow \int_0^{\infty} \sqrt{1+t^2} dt = \lim_{2x \rightarrow \infty} \int_0^{2x} \sqrt{1+t^2} dt$$

Def 6.16 (b, f)

Tip: Splits $\frac{1}{x(x-1)}$ op in partiele breuken

$$b) \int_2^{\infty} \frac{dx}{x(x-1)} \rightarrow \frac{A}{x} + \frac{B}{x-1}$$

$$1 = Ax - A + Bx$$

$$\begin{cases} 0 = A + B \\ 1 = -A \end{cases} \Leftrightarrow \begin{cases} B = 1 \\ A = -1 \end{cases}$$

$$= \int_2^{\infty} \frac{-1}{x} + \frac{1}{x-1} dx = - \int_2^{\infty} \frac{1}{x} dx + \int_2^{\infty} \frac{1}{x-1} dx$$

$$= - \left[\ln|x| \right]_2^{\infty} + \left[\ln|x-1| \right]_2^{\infty}$$

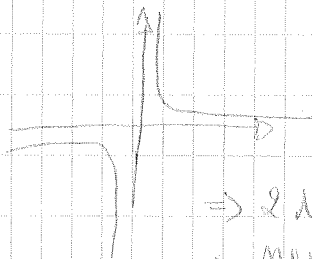
$$= -\cancel{\ln \infty} + \ln 2 + \underbrace{\ln|\infty-1|}_{=\ln \infty} - \underbrace{\ln 1}_0$$

$$= \ln 2$$

$$g) \int_{-1}^2 \left(\frac{1}{x} \right) dx = \text{[scribbled out]}$$

$$\hookrightarrow \int_{-1}^a \frac{1}{x} dx + \int_b^2 \frac{1}{x} dx$$

$$= \lim_{a \rightarrow 0} \int_{-1}^a \frac{1}{x} dx + \lim_{b \rightarrow 0} \int_b^2 \frac{1}{x} dx$$



$\Rightarrow$ 2 versch. lim nemen

$$\begin{aligned} &= \lim_{a \rightarrow 0} \left[\ln|x| \right]_{-1}^a + \lim_{b \rightarrow 0} \left[\ln|x| \right]_b^2 \\ &= \lim_{a \rightarrow 0} (\ln|a| - \ln|1|) + \lim_{b \rightarrow 0} (\ln|2| - \ln|b|) \\ &= \lim_{a \rightarrow 0} (\ln|a| - \ln 1) + \lim_{b \rightarrow 0} (\ln 2 - \ln|b|) \end{aligned}$$

$$= (\ln 0 - \ln 1) + (\ln 2 - \ln 0) = \infty$$

$\Rightarrow$ divergent

oef 6.20(a,f)

a) $f(x) = 1 - x^2$, $[a, b] = [0, 1]$, $n = 8$

* Trapeziumregel T_n
voor gfg. n

* Afschatting integr. fout
naar stelling 6.6.3

* $T_8 = \frac{1-0}{2 \cdot 8} (f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_7) + f(x_8))$ * waarde n zodanig

$\left| \int_a^b f(x) dx - T_n \right| < 10^{-5}$

~~$= \frac{1}{16} (1 + 2 \cdot 0 + 2 \cdot (-3) + 2 \cdot (-7) + 2 \cdot (-15) + 2 \cdot (-24) + 2 \cdot (-35) + 2 \cdot (-48) + 2 \cdot (-63))$~~

~~$= \frac{1}{16} (1 + 0 - 6 - 14 - 30 - 48 - 70 - 96 - 126)$~~

~~$= \frac{1}{16} (-391)$~~

$= \frac{1}{16} \left[\overset{x_0}{0} + \overset{x_1}{2 \cdot \frac{1}{8}} + \overset{x_2}{2 \cdot \frac{1}{8}} + \overset{x_3}{3 \cdot \frac{1}{8}} + \overset{x_4}{4 \cdot \frac{1}{8}} + \overset{x_5}{5 \cdot \frac{1}{8}} + \overset{x_6}{6 \cdot \frac{1}{8}} + \overset{x_7}{7 \cdot \frac{1}{8}} + \overset{x_8}{8 \cdot \frac{1}{8}} \right]$

~~$= \frac{1}{16} \left[\frac{1}{8} + \frac{2}{8} + \frac{3}{8} + \frac{4}{8} + \frac{5}{8} + \frac{6}{8} + \frac{7}{8} + \frac{8}{8} \right]$~~

~~$= \frac{1}{16} [1 + 1 + 1 + 1 + 1 + 1 + 1 + 1] = \frac{1}{16} [8] = \frac{1}{2}$~~

$= \frac{1}{16} \left[1 + \left(1 - \frac{1}{64}\right) + \left(1 - \frac{4}{64}\right) + \left(1 - \frac{9}{64}\right) + \left(1 - \frac{16}{64}\right) + \left(1 - \frac{25}{64}\right) + \left(1 - \frac{36}{64}\right) + \left(1 - \frac{49}{64}\right) + \left(1 - \frac{64}{64}\right) \right]$

$= \frac{1}{16} \left[1 + \frac{63}{64} + \frac{15}{64} + \frac{55}{64} + \frac{3}{4} + \frac{39}{64} + \frac{7}{16} + \frac{15}{64} \right]$

voorbereiding wiskunde-oefeningen

oefenzitting 11 ~ 13 | 11 | 2019

oef 7.3 (c)

* d) opp v gebied tn $x = |y|$ en $x = 2 - y^2$

$$S = \int_a^b (2 - y^2 - |y|) dy =$$

oef 7.5

a) $A_m = \int_1^m \frac{1}{x^{4/3}} dx =$

voorbereid.

Wiskunde 1 - oefeningen

oefenzitting 11-13/11/2019

oef. 7.3 (c)

c) opp tm $x = |y|$ en $x = 2 - y^2 \rightarrow x = |y| \begin{cases} x = -y \\ x = y \end{cases}$

* $x = |y| \Rightarrow |x| = y \quad \forall x \in [0, \infty[$ te oplossen, dan...

$y = \sqrt{2-x}$

* $x = \sqrt{2-x}$

$\Leftrightarrow x^2 = 2-x \Leftrightarrow x^2 + x - 2 = 0$

$D = 1 - 4 \cdot 1 \cdot (-2) = 9$

$x = \frac{-1 \pm 3}{2} = 1 \text{ of } -2 \text{ (vw)}$

* $|y| = 2 - y^2$

$\Leftrightarrow y = 2 - y^2 \quad \forall y \in [0, \infty[$

$\Leftrightarrow y^2 + y - 2 = 0$

$\Delta > D = 9$

$\Delta > y = \frac{-1 \pm 3}{2} = 1 \text{ of } -2$

~~is niet mogelijk~~

$\Delta > x = |1| \text{ of } x = |-2|$

$\Delta > x = 1 \text{ of } 2$

~~$y = 2 - y^2$
 $\Leftrightarrow y^2 + y - 2 = 0$~~

~~$\Delta > y = \frac{-1 \pm 3}{2} = 2 \text{ of } -1$
 $\Delta > \text{niet mogelijk}$~~

opp = $\int_1^2 x^2 + x - 2 \, dx = \left[\frac{x^3}{3} + \frac{x^2}{2} - 2x \right]_1^2$

$= \left(\frac{8}{3} + \frac{4}{2} - 4 \right) - \left(\frac{1}{3} + \frac{1}{2} - 2 \right) = \frac{8}{3} + 2 - 4 - \frac{1}{3} + \frac{1}{2} - 2$

$= \frac{4}{3} + \frac{1}{2} =$

$$\int x^m = \frac{x^{m+1}}{m+1} + C$$

ex 7.5

$$\begin{aligned} \text{a) } A_m &= \int_1^m \frac{1}{x^{4/3}} dx = \int_1^m x^{-4/3} dx = \left[\frac{x^{-1/3}}{-1/3} \right]_1^m \\ &= \left[-3x^{-1/3} \right]_1^m = -3m^{-1/3} + 3 = \frac{-3}{m^{1/3}} + 3 = 3 \left(1 - \frac{1}{m^{1/3}} \right) \end{aligned}$$

$$\text{b) } \lim_{m \rightarrow \infty} A_m = 3$$

ex 7.9 (a, c)

$$\text{a) } \begin{cases} x = e^t \cos t \\ y = e^t \sin t \end{cases} \quad 0 \leq t \leq \pi/4$$

$$\begin{aligned} \text{length } K &= \int_{\theta_1}^{\theta_2} \sqrt{(f'(\theta))^2 + (g'(\theta))^2} d\theta \\ &= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \end{aligned}$$

~~$$\int_0^{\pi/4} \sqrt{(e^t \cos t - e^t \sin t)^2 + (e^t \sin t + e^t \cos t)^2} dt$$~~

$$\begin{aligned} &\int_0^{\pi/4} \sqrt{(e^t \cos t + e^t (-\sin t))^2 + (e^t \sin t + e^t \cos t)^2} dt \\ &= \int_0^{\pi/4} \sqrt{(e^{2t} \cos^2 t - 2(e^{2t} \cos t \sin t) + e^{2t} \sin^2 t) + (e^{2t} \sin^2 t + 2(e^{2t} \cos t \sin t) + e^{2t} \cos^2 t)} dt \\ &= \int_0^{\pi/4} \sqrt{(e^{2t} \cos^2 t + e^{2t} \sin^2 t + e^{2t} \sin^2 t + e^{2t} \cos^2 t)} dt \\ &= \int_0^{\pi/4} \sqrt{(e^{2t}) (\underbrace{\cos^2 t + \sin^2 t}_1 + \underbrace{\sin^2 t + \cos^2 t}_1)} dt \\ &= \int_0^{\pi/4} \sqrt{(e^{2t}) \cdot 2} dt = \sqrt{2} \left(\int_0^{\pi/4} \sqrt{e^{2t}} dt \right) = \sqrt{2} \int_0^{\pi/4} e^t dt \\ &= \sqrt{2} [e^t]_0^{\pi/4} = \sqrt{2} (e^{\pi/4} - e^0) = \sqrt{2} (e^{\pi/4} - 1) \end{aligned}$$

verwolg 7.9

c) $y = \ln x \quad 1 \leq x \leq 4$

$$\int_a^b \sqrt{1 + [f'(x)]^2} dx$$

$$\text{Lengte kromme} = \int_1^4 \sqrt{1 + \frac{1}{x^2}} dx$$

oef. 7.10(c)

c) $r = 3\theta^2 \quad 1 \leq \theta \leq 2$

$$f(\theta) = 3\theta^2 \quad \text{en} \quad f'(\theta) = 6\theta$$

$$(\rightarrow x = 3\theta^2 \cos \theta \quad \text{en} \quad y = 3\theta^2 \sin \theta)$$

$$\text{Lengte} = \int_1^2 \sqrt{(3\theta^2)^2 + (6\theta)^2} d\theta = \int_1^2 \sqrt{9\theta^4 + 36\theta^2} d\theta$$

$$= \int_1^2 \sqrt{9\theta^2(\theta^2 + 4)} d\theta = 3\theta \int_1^2 \sqrt{\theta^2 + 4} d\theta$$

$$= 3\theta \int_1^2 (\theta^2 + 4)^{1/2} d\theta$$

oef. 7.14

opp. gebied kwadrant I ingesloten door r^2 en spiraal $r = \theta$
en waerstralen $\theta = \pi/6$ en $\theta = \pi/3$

* kwadrant I $\Rightarrow 0 \leq \theta \leq \frac{\pi}{2}$

$$\frac{1}{2} \int_{\pi/6}^{\pi/3} [\theta]^2 d\theta = \frac{1}{2} \left[\frac{\theta^3}{3} \right]_{\pi/6}^{\pi/3} = \frac{1}{2} \left(\frac{\pi^3/27}{3} - \frac{\pi^3/216}{3} \right)$$

$$= 0,167 = \frac{7}{1296} \pi^3$$

$$\left(\frac{e^{-2x}}{e} \right)' = \frac{1-2x e^{-2x}}{-2x}$$

Def 7.17(f)

f) $y = e^{-x} \quad 0 \leq x < \infty$

$$V = \pi \int_0^{\infty} e^{-2x} dx = \pi \left[\frac{e^{-2x}}{-2x} \right]_0^{\infty} = \frac{\pi}{2}$$

Def 7.19

→ volume $y = \sqrt{x}$, $0 \leq x \leq 4$ rond x-ax

$$V = \pi \int_0^4 (\sqrt{x})^2 dx = \pi \left[\frac{x^2}{2} \right]_0^4 = \pi \left(\frac{16}{2} - 0 \right) = 8\pi$$

→ rond y-ax $\Rightarrow x = y^2$

$$V = \pi \int_0^4 y^4 dy = \pi \left[\frac{y^5}{5} \right]_0^4 = \pi \frac{1024}{5}$$

13/11/19

Wiskunde 1 - oefeningen

opgave 11

Oef. 7.2

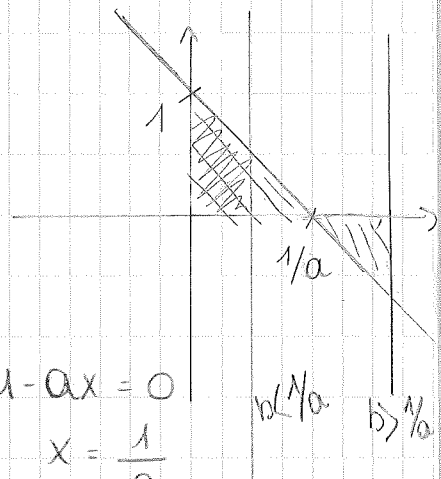
Tip: Maak een onderscheid tuss. gevallen $b < \frac{1}{a}$ en $b > \frac{1}{a}$. Maak in elk v. beide gevallen een schets

$$y = 1 - ax$$

$$x = a$$

$$x = 0$$

$$x = b$$

2 gevallen: $b < \frac{1}{a}$ $b > \frac{1}{a}$ 

$$\text{opp} = \int_0^b (1 - ax) dx$$

$$\text{mijpunt: } y = 1 - ax = 0 \\ x = \frac{1}{a}$$

$$\textcircled{1} b < \frac{1}{a} : \int_0^b (1 - ax) dx = \left[x - \frac{ax^2}{2} \right]_0^b \\ = b + \frac{ab^2}{2}$$

$$\textcircled{2} b > \frac{1}{a} : \int_0^{1/a} (1 - ax) dx + \int_{1/a}^b (-1 + ax) dx$$

$$= \left[x - \frac{ax^2}{2} \right]_0^{1/a} + \left[-x + \frac{ax^2}{2} \right]_{1/a}^b$$

$$= \left[\frac{1}{a} - \frac{a \frac{1}{a^2}}{2} \right] + \left[-b + \frac{ab^2}{2} + \frac{1}{a} - \frac{a \frac{1}{a^2}}{2} \right]$$

$$= \frac{1}{a} - \frac{1}{2a} - b + \frac{ab^2}{2} + \frac{1}{a} - \frac{1}{2a}$$

$$= \frac{2}{a} - \frac{2}{2a} - b + \frac{ab^2}{2} = \frac{4 - 2}{2a} - b + \frac{ab^2}{2}$$

$$= \frac{2}{2a} - b + \frac{ab^2}{2} = \frac{1}{a} - b + \frac{ab^2}{2}$$

oef. 7.6

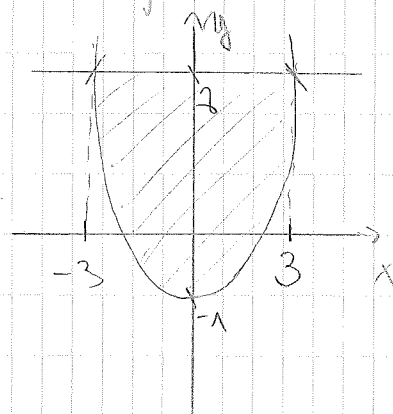
Tip: Maak een schets. Het is gemakkelijker te integreren over y .

Bepaal $y=c$

$$y = x^2 - 1$$

$$y = 8$$

} in 2 delen met gelijke opp.



* snijpunt: $8 = x^2 - 1$

$$9 = x^2$$

$$\pm 3 = x$$

* tot. opp. = $\int_{-3}^3 (8 - x^2 + 1) dx = \int_{-3}^3 (9 - x^2) dx = \left[9x - \frac{x^3}{3} \right]_{-3}^3 = 36$

$\rightarrow$ helft opp. = 18

$\rightarrow$ de helft v.d. helft $x = \pm \sqrt{y+1}$

* helft opp. = $\int_{-1}^c \sqrt{y+1} dy = 18 = \int_{-1}^c (y+1)^{1/2} dy$ $\begin{cases} y+1=t \\ dy=dt \end{cases}$

$$= \int_{-1}^c t^{1/2} dy = \left[(y+1)^{3/2} \cdot \frac{2}{3} \right]_{-1}^c$$

$$= \left((c+1)^{3/2} \cdot \frac{2}{3} - 0 \right) = 9$$

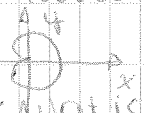
$$\Rightarrow (c+1)^{3/2} = 9 \cdot \frac{3}{2} = \frac{27}{2}$$

$$\Rightarrow c+1 = \left(\frac{27}{2} \right)^{2/3} \Rightarrow c = \left(\frac{27}{2} \right)^{2/3} - 1$$

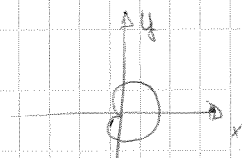
$$c = 4,67$$

wereld
13/11

oef. 7.11(a)

Tip: zie paragraaf 7.3. Deze kromme wordt al eens getekend in oef. 3.16 (b): 
 let op: integratie w over θ (niet x of y): wat is θ en zijn naar grenzen? $\cos 2\theta = 2\cos^2\theta - 1$

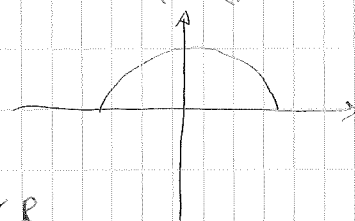
a) $r = 2(1 + \cos \theta)$



$$\begin{aligned} \text{opp} &= \frac{1}{2} \int_a^b (f(\theta))^2 d\theta = \frac{1}{2} \int_0^{2\pi} (2 + 2\cos\theta)^2 d\theta \\ &= \frac{1}{2} \int_0^{2\pi} (4 + 8\cos\theta + 4\cos^2\theta) d\theta \\ &= \frac{1}{2} \int_0^{2\pi} 4 d\theta + \frac{1}{2} \int_0^{2\pi} 8\cos\theta d\theta + \frac{1}{2} \int_0^{2\pi} 4\cos^2\theta d\theta \\ &= 2[\theta]_0^{2\pi} + 4[\sin\theta]_0^{2\pi} + 2 \int_0^{2\pi} \frac{\cos 2\theta + 1}{2} d\theta \\ &= 4\pi + \int_0^{2\pi} \cos 2\theta d\theta + 2 \int_0^{2\pi} \frac{1}{2} d\theta \\ &= 4\pi + \frac{1}{2} [\sin 2\theta]_0^{2\pi} + [\theta]_0^{2\pi} = 4\pi + 2\pi = 6\pi \end{aligned}$$

oef. 7.15

$$S = 2\pi \int_a^b f(x) [1 + (f'(x))^2]^{1/2} dx \quad \text{met } a \leq x \leq b$$



Bol: halve cirkel met straal R :

$$y = f(x) = \sqrt{R^2 - x^2} \quad \text{met } -R \leq x \leq R$$

$$\begin{aligned} S &= 2\pi \int_{-R}^R \sqrt{R^2 - x^2} \left[1 + \left(\frac{x}{\sqrt{R^2 - x^2}} \right)^2 \right]^{1/2} dx \\ &= 2\pi \int_{-R}^R \sqrt{R^2 - x^2} \left[1 + \frac{x^2}{R^2 - x^2} \right]^{1/2} dx \\ &= 2\pi \int_{-R}^R \sqrt{R^2 - x^2} \left[\frac{R^2 - x^2 + x^2}{R^2 - x^2} \right]^{1/2} dx \\ &= 2\pi \int_{-R}^R \sqrt{R^2 - x^2} \sqrt{\frac{R^2}{R^2 - x^2}} dx = 2\pi \int_{-R}^R \sqrt{R^2} \frac{\sqrt{R^2}}{\sqrt{R^2 - x^2}} dx \end{aligned}$$

$$= 2\pi \int_{-R}^R R \, dx = 2\pi [Rx]_{-R}^R = 2\pi (R^2 + R^2) = 2\pi 2R^2 = 4\pi R^2$$

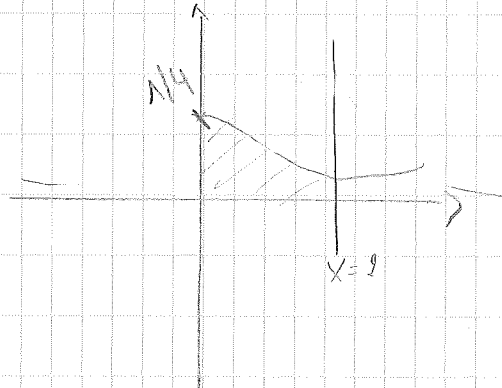
oef 7 18 Tip: Maak een schets. Geg: $\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$

$$y = \frac{1}{4+x^2}$$

$$x=0$$

$$y=0$$

$$x=2$$



$$a) \text{opp} = \int_0^2 \frac{1}{4+x^2} dx$$

$$= \frac{1}{2} \left[\arctan\left(\frac{x}{2}\right) \right]_0^2 = \frac{1}{2} \arctan 1 - \frac{1}{2} \arctan 0$$

$$= \frac{1}{2} \cdot \frac{\pi}{4} = \frac{\pi}{8}$$

b) wervelen rond y-as. $y(4+x^2) = 1$

$$4+x^2 = \frac{1}{y}$$

$$V = \pi \int_a^b [f(x)]^2 dx$$

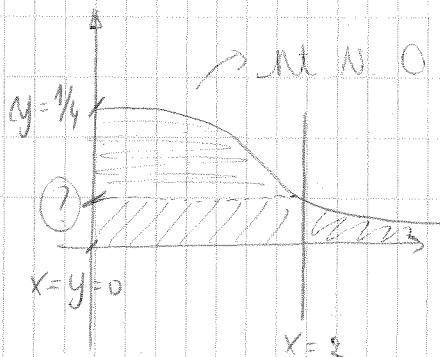
$$x^2 = \frac{1}{y} - 4 \Rightarrow x = \pm \sqrt{\frac{1-4y}{y}}$$

$$= \pi \int_0^2 \frac{1-4y}{y} dy = \pi \int_0^2 \frac{1}{y} dy - \pi \int_0^2 \frac{4y}{y} dy$$

$$= \pi [\ln |y|]_0^2 - \pi [4y]_0^2$$

$$= \pi \ln 2 - \underbrace{\pi \ln 0}_{=1\pi} - \pi 8 + \underbrace{\pi 4 \cdot 0}_{=0}$$

$$= \pi \ln 2 - \pi - 8\pi = \pi \ln 2 - 9\pi$$



→ nu v. 0 tot $\frac{1}{4}$ integreer → dan heb je ook

$$x=2 \Rightarrow \frac{1}{4+4} = \frac{1}{8}$$

nu v. 0 tot ② → 2 en dan

nu x=0
↑
1/4
②

vervolg
oef 7.13

$$\begin{aligned} & \pi \int_0^{1/8} 2^x dy + \pi \int_{1/8}^{1/4} \frac{1-4y}{y} dy = \pi [4x]_0^{1/8} + \pi [\ln|y| - 4y]_{1/8}^{1/4} \\ &= \pi \left(\frac{1}{2} - 0 \right) + \pi \left(\ln \frac{1}{4} - 1 - \left(\ln \frac{1}{8} - \frac{1}{2} \right) \right) \\ &= \frac{1}{2} \pi + \pi \left(\ln \frac{1}{4} - \ln \frac{1}{8} - 1 + \frac{1}{2} \right) \\ &= \cancel{\frac{1}{2} \pi} + \cancel{\frac{1}{2} \pi} - \pi + \pi \left(\underbrace{\ln \frac{1/4}{1/8}}_{= \ln 2} \right) = \pi \ln 2 \end{aligned}$$

oef 7.29

Tip: zie paragraaf 7.6.2
voor c): de kans dat x tn a en b
ligt is geg door $\int_a^b p(x) dx$

voorbereid

Wiskunde 1 - oefeningen

oefenzitting 12-15/11/2019

oef 85 (a, b, f, m)

a) $\int (3x+1)^p dx$

$\hookrightarrow$ voor $p = -1 \Rightarrow$ subst: $3x+1 = t$
 $3dx = dt$

$$\Leftrightarrow \frac{1}{3} \int \frac{1}{t} dt = \frac{1}{3} \ln |3x+1| + C$$

$\hookrightarrow$ voor $p \neq -1 \Rightarrow \frac{(3x+1)^{p+1}}{(p+1)3} + C$

b) $\int_0^2 x \sqrt{4-x^2} dx = \int_0^2 \frac{1}{2} \cdot \sqrt{4-t^2} dt = \frac{1}{2} \int_0^2 \sqrt{4-t^2} dt = \int_0^1 \sqrt{1-\frac{t^2}{4}} dt$

$$x^2 = t$$

$$2x dx = dt$$

$$4-x^2 = t$$

$$-2x dx = dt$$

$$\frac{t}{2} = u$$

$$\frac{1}{2} dt = du$$

$$= \frac{1}{2} \int_0^2 \sqrt{1-u^2} du =$$

$$= -\frac{1}{2} \int_0^2 \sqrt{t} dt = -\frac{1}{2} \left[\frac{(4-x^2)^{3/2}}{3/2} \right]_0^2 = -\frac{1}{2} \left(\frac{(0)^{3/2}}{3/2} - \frac{(4)^{3/2}}{3/2} \right) = \frac{+1}{2} + \frac{8 \cdot 2}{3} = \frac{8}{3}$$

f) $\int e^{\sin x} \cos x dx = \int e^t dt = e^{\sin x} + C$

$$\sin x = t$$

$$\cos x dx = dt$$

m) $\int \tan x dx = \int \frac{\sin x}{\cos x} dx = -\int \frac{1}{t} dt = -\ln |\cos x| + C$

$$\cos x = t$$

$$-\sin x dx = dt$$

Def 8.10 (a, b)

$$\int f \cdot g' = f \cdot g - \int f' \cdot g$$

a) $\int x \sin x \, dx$

$\hookrightarrow f(x) = x$

$g'(x) = \sin x \Rightarrow g(x) = -\cos x$

$$= -x \cos x - \int -\cos x \, dx = -x \cos x + \sin x + C$$

b) $\int x^p \ln x \, dx$

$\hookrightarrow \text{als } p = -1: \int \frac{1}{x} \ln x \, dx$

$\hookrightarrow f(x) = \ln x$

$g'(x) = 1/x \Rightarrow g(x) = \ln x$

$$= \ln^2 x - \int \frac{1}{x} \ln x \, dx$$

$$\Rightarrow I = \ln^2 x - I \Leftrightarrow 2I = \ln^2 x \Leftrightarrow I = \frac{\ln^2 x}{2} + C$$

$\hookrightarrow \text{als } p \neq -1: \int x^p \ln x \, dx$

$\hookrightarrow f(x) = \ln x$

$g'(x) = x^p \Rightarrow g(x) = \frac{x^{p+1}}{p+1}$

$$= \ln x \cdot \frac{x^{p+1}}{p+1} - \int \frac{1}{x} \cdot \frac{x^{p+1}}{p+1} \, dx$$

$$= \ln x \cdot \frac{x^{p+1}}{p+1} - \frac{1}{p+1} \int \underbrace{\frac{1}{x} x^{p+1}}_{= x^p} \, dx$$

$$= \ln x \cdot \frac{x^{p+1}}{p+1} - \frac{1}{p+1} \cdot \frac{x^{p+1}}{p+1} + C$$

$$= \frac{x^{p+1}}{p+1} \left(\ln x - \frac{1}{p+1} \right) + C$$

Def 8.11 (d)

$$d) \int_0^{\infty} x e^{-2x} dx$$

$$\hookrightarrow f(x) = x$$

$$g'(x) = e^{-2x} \Rightarrow g(x) = \frac{e^{-2x}}{-2}$$

$$= \left[\frac{e^{-2x}}{-2} \right]_0^{\infty} - \frac{1}{-2} \int_0^{\infty} \frac{e^{-2x}}{x} dx$$

$$= -\frac{1}{2} - \frac{1}{2} \int_0^{\infty} \frac{e^{-2x}}{x} dx$$

15/11/2019

Wiskunde 1 - oefeningen

oefenzitting 12

oef. 8.1 (a, e)

Tip: d) let op waar $n=0$

$$d) \int_0^{\pi/2} \cos^n nx \, dx \quad \text{met } n \in \mathbb{N}$$

$$\begin{aligned} \cos 2x &= 2\cos^2 x - 1 \\ &= \cos^2 x - \sin^2 x \end{aligned}$$

$$= \int_0^{\pi/2} \frac{\cos 2nx - 1}{2} \, dx = \frac{1}{2} \int_0^{\pi/2} (\cos 2nx - 1) \, dx$$

$$= \frac{1}{2} \left[\frac{\sin 2nx}{2n} - x \right]_0^{\pi/2} = \frac{1}{2} \left(\frac{\sin 2n \frac{\pi}{2}}{2n} - \frac{\pi}{2} - \frac{\sin 2n \cdot 0}{2n} + 0 \right)$$

$$= \frac{1}{2} \left(\frac{\sin n}{2n} - \frac{\pi}{2} \right) = \frac{\sin n}{2n} - \frac{\pi}{4} \quad (\text{als } n \neq 0)$$

$$* \int_0^{\pi/2} \cos^2 0 \, dx = \int_0^{\pi/2} 1 \, dx = [x]_0^{\pi/2} = \frac{\pi}{2} \quad (\text{als } n=0)$$

$$e) \int_0^{\pi} \sin x \cos 2x \, dx = \int_0^{\pi} \sin x (2\cos^2 x - 1) \, dx$$

$$= \int_0^{\pi} 2\sin x \cos^2 x - \sin x \, dx = 2 \int_0^{\pi} \sin x \cos^2 x \, dx - \int_0^{\pi} \sin x \, dx$$

$$\cos x = t$$

$$-\sin x \, dx = dt$$

$$= -2 \int_0^{\pi} t^2 \, dt - \int_0^{\pi} \sin x \, dx = -2 \left[\frac{t^3}{3} \right]_0^{\pi} - [-\cos x]_0^{\pi}$$

$$= -2 \left[\frac{\cos^3 x}{3} \right]_0^{\pi} - [-\cos x]_0^{\pi} = -2 \left(\frac{\cos^3 \pi}{3} - \frac{\cos^3 0}{3} \right) - (-\cos \pi + \cos 0)$$

$$= -2 \left(-\frac{1}{3} - \frac{1}{3} \right) - (-1 + 1) = -2 \left(-\frac{2}{3} \right) - 2 = \frac{4}{3} - 2 = -\frac{2}{3}$$

ex. 8.5 (a, b)

$$e) \int e^x (1+e^x)^{1/2} dx = \int t^{1/2} dt = \frac{(1+e^x)^{3/2}}{3/2} + C$$

$1+e^x = t$
 $e^x dx = dt$

$$= \frac{2}{3} (1+e^x)^{3/2} + C$$

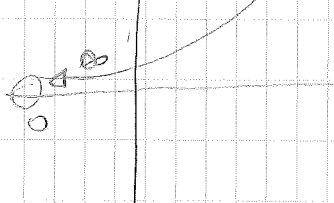
$$h) \int \frac{dx}{x \ln x} = \int \frac{1}{t} dt = \ln |\ln x| + C$$

$\ln x = t$
 $\frac{1}{x} dx = dt$

ex. 8.7 (b)

$$b) \int_0^{\infty} x e^{-x^2} dx = \int_{\infty}^0 \frac{-1}{2} dt = - \int_{\infty}^0 \frac{1}{2} dt = \int_0^{\infty} \frac{1}{2} dt = \left[\frac{1}{2} x \right]_0^{\infty} = \frac{1}{2}$$

$e^{-x^2} = t$
 $-2x e^{-x^2} dx = dt$
 $x \rightarrow 0 \text{ dan } t \rightarrow 1$
 $x \rightarrow \infty \text{ dan } t \rightarrow 0$



ex. 8.10 (f) Tip: 2 x part. integr. temp

$$f) \int e^{-2x} \cos x dx$$

$\hookrightarrow f(x) = e^{-2x}$
 $g'(x) = \cos x \Rightarrow g(x) = \sin x$

$$= \sin x \cdot e^{-2x} - \int -2e^{-2x} \cdot \sin x dx$$

$$= \sin x \cdot e^{-2x} + 2 \int e^{-2x} \cdot \sin x dx$$

$\hookrightarrow f(x) = e^{-2x}$

$g(x) = -\cos x$

$$= \sin x \cdot e^{-2x} - 2 \cos x \cdot e^{-2x} - 2 \int -2e^{-2x} (-\cos x) dx$$

$$= \sin x \cdot e^{-2x} - 2 \cos x \cdot e^{-2x} - 4 \int e^{-2x} \cos x dx$$

$= I \text{ (opposite)}$

$$I = \sin x \cdot e^{-2x} - 2 \cos x \cdot e^{-2x} - 4 I$$

$$5I = \sin x (e^{-2x}) - 2 \cos x (e^{-2x}) \Rightarrow I = \frac{1}{5} e^{-2x} \sin x - \frac{2}{5} e^{-2x} \cos x + C$$

oef. 8.11 (a)

a) $\int_0^1 \ln x \, dx$

$\hookrightarrow f(x) = \ln x$

$g'(x) = 1 \Rightarrow g(x) = x$

$$= \left[x \ln x \right]_0^1 - \int_0^1 \frac{1}{x} \cdot x \, dx = \left[x \ln x \right]_0^1 - \left[x \right]_0^1$$

$$= \ln 1 - 0 - 1 + 0 = -1$$

oef. 8.14

$$\int_0^{\infty} x^n e^{-x} \, dx = n! \quad \text{voor elke } n \in \mathbb{N}$$

"1 · 2 · 3 · ... · (n-1) · n"

• Basisstap

$n = 0$

RL $\Rightarrow 0' = 1$

LL $\Rightarrow \int_0^{\infty} x^0 e^{-x} \, dx = \int_0^{\infty} 1 \cdot e^{-x} \, dx = \int_0^{\infty} e^{-x} \, dx = \left[-e^{-x} \right]_0^{\infty} = 1$

$\Rightarrow$ LL = RL dus basisstap is waar

• Inductiestap $\rightarrow$ I.H. : $\int_0^{\infty} x^n e^{-x} \, dx = n!$ voor $n \geq 0$
 $\hookrightarrow$ we nemen aan dat dit klopt

$\rightarrow$ T.B. : nu gaan we bewijzen dat het ook klopt voor $n+1$.

$\rightarrow$ Bewijs: $\int_0^{\infty} x^{n+1} e^{-x} \, dx = (n+1)!$

RL $\Rightarrow (n+1)! = (n+1)(n)(n-1) \dots 2 \cdot 1$

LL $\Rightarrow \int_0^{\infty} x^{n+1} e^{-x} \, dx = \int_0^{\infty} x^1 \cdot \underbrace{x^n \cdot e^{-x}} \, dx$

$\hookrightarrow f(x) = x^{n+1}$

$g'(x) = e^{-x} \Rightarrow -e^{-x} = g(x)$

$$\begin{aligned}
 &= \lim_{a \rightarrow \infty} \left[x^{n+1} \cdot (-e^{-x}) \right]_0^a - \int_0^a (n+1) \cdot x^{(n+1)-1} \cdot (-e^{-x}) dx \\
 &\text{onbepaald} \leftarrow \infty \cdot 0 \\
 (\ast) &= \lim_{a \rightarrow \infty} \underbrace{\left[x^{n+1} \cdot (-e^{-x}) \right]_0^a}_{=0} + (n+1) \underbrace{\int_0^a x^n e^{-x} dx}_{n!} \\
 &= (n+1) \cdot n! = (n+1)! \\
 &\Rightarrow LL = RL
 \end{aligned}$$

* conclusie: Omdat de basisstap & inductiestap bewezen zijn, geldt $\int_0^\infty x^n e^{-x} dx = n!$ voor elke $n \geq 0$, vanwege het principe van volledige inductie.

$$\begin{aligned}
 (\ast) \quad \lim_{a \rightarrow \infty} (a^{n+1} \cdot (-e^{-a})) &= \lim_{a \rightarrow \infty} \frac{a^{n+1}}{-e^a} \\
 \stackrel{\text{L'H}}{=} \lim_{a \rightarrow \infty} \frac{(n+1)a^n}{a} &= \dots (n \text{ keer H}) = 0
 \end{aligned}$$

oef. 8.15 (c, e, j.)

c) $\int \frac{x}{(x+1)^2 (x-1)^2} dx$

$$\hookrightarrow \frac{x}{(x+1)^2 (x-1)^2} = \frac{Ax+B}{(x+1)^2} + \frac{Cx+D}{(x-1)^2}$$

$x^2+2x+1 \quad \quad x^2-2x+1$

$$x = Ax^3 - 2Ax^2 + Ax + Bx^2 - 2Bx + B + Cx^3 + 2Cx^2 + Cx + Dx^2 + 2Dx + D$$

$$\begin{cases} 0 = A + C \\ 0 = -2A + B + 2C + D \\ 1 = A - 2B + C + 2D \\ 0 = B + D \end{cases} \Leftrightarrow \begin{cases} A = 0 \\ B = -1/4 \\ C = 0 \\ D = 1/4 \end{cases}$$

$$= \int \frac{-1/4}{(x+1)^2} dx + \int \frac{1/4}{(x-1)^2} dx = \frac{-1}{4} \int \frac{1}{(x+1)^2} dx + \frac{1}{4} \int \frac{1}{(x-1)^2} dx$$

$$= \frac{-1}{4} \int \frac{1}{t^2} dt + \frac{1}{4} \int \frac{1}{u^2} du$$

$x+1=t \quad \quad x-1=u$
 $dx=dt \quad \quad dx=du$

$$= \frac{-1}{4} \cdot \left(\frac{t^{-1}}{-1} \right) + \frac{1}{4} \left(\frac{u^{-1}}{-1} \right) + C = \frac{1}{4} \frac{1}{x+1} - \frac{1}{4} \frac{1}{x-1} + C$$

$$\ln x^2 = 2 \ln x$$

$$e) \int \frac{dx}{x(x^2+x+1)}$$

$$\hookrightarrow \frac{1}{x(x^2+x+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+x+1}$$

$$1 = Ax^2 + Ax + A + Bx^2 + Cx$$

$$\begin{cases} 0 = A+B \\ 0 = A+B \\ 1 = A \end{cases} \Leftrightarrow \begin{cases} A = 1 \\ B = -1 \\ C = -1 \end{cases}$$

$$= \int \frac{1}{x} dx + \int \frac{-x-1}{x^2+x+1} dx = \int \frac{1}{x} dx - \int \frac{x+1}{x^2+x+1} dx$$

$$\begin{cases} x^2+x+1 = t \\ (2x+1)dx = dt \end{cases}$$

$$= \int \frac{1}{x} dx - \left(\frac{1}{2} \int \frac{2x+1}{x^2+x+1} dx + \int \frac{1/2}{(x+\frac{1}{2})^2 + \frac{3}{4}} dx \right)$$

$$= \int \frac{1}{x} dx - \left(\frac{1}{2} \int \frac{1}{t} dt + \frac{1}{2} \int \frac{1}{(x+\frac{1}{2})^2 + \frac{3}{4}} dx \right)$$

$$= \ln|x| - \left(\frac{1}{2} \ln|t| + \frac{1}{2} \cdot \frac{4^2}{3} \int \frac{1}{\frac{4(x+\frac{1}{2})^2}{3} + 1} dx \right)$$

$$\begin{cases} \frac{2(x+\frac{1}{2})}{\sqrt{3}} = u \\ \frac{2}{\sqrt{3}} dx = du \end{cases}$$

$$= \ln|x| - \frac{1}{2} \ln|t| - \frac{2}{3} \cdot \frac{\sqrt{3}}{2} \int \frac{du}{u^2+1}$$

$$= \ln|x| - \frac{1}{2} \ln|x^2+x+1| - \frac{\sqrt{3}}{3} \operatorname{Bgtan}\left(\frac{2x+1}{\sqrt{3}}\right) + C$$

$$= \frac{1}{2} \ln \left| \frac{x^2}{x^2+x+1} \right| - \frac{\sqrt{3}}{3} \operatorname{Bgtan}\left(\frac{2x+1}{\sqrt{3}}\right) + C$$

$= 1/\sqrt{3}$

$$j) \int \frac{x^3}{x^3-1} dx = \int \frac{x^3-1+1}{x^3-1} dx = \int \frac{x^3-1}{x^3-1} dx + \int \frac{1}{x^3-1} dx$$

$$= \int 1 dx + \int \frac{1}{(x-1)(x^2+x+1)} dx$$

$$\hookrightarrow \frac{1}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$

$$1 = Ax^2 + Ax + A + Bx^2 + Bx + Cx - C$$

$$\begin{cases} 0 = A+B \\ 0 = A-B+C \\ 1 = A-C \end{cases} \Leftrightarrow \begin{cases} A = 1/3 \\ B = -1/3 \\ C = -2/3 \end{cases}$$

$$= x + \int \frac{1/3}{x-1} dx + \int \frac{-1/3 x - 2/3}{x^2+x+1} dx = x + \frac{1}{3} \int \frac{1}{x-1} dx - \frac{1}{3} \int \frac{x+2}{x^2+x+1} dx$$

$$= x + \frac{1}{3} \ln|x-1| - \frac{1}{3} \int \frac{x+2}{x^2+x+1} dx$$

$$\begin{aligned} &\rightarrow p. 215 \Rightarrow \left(b - \frac{ap}{2}\right) \\ &\left(2 - \frac{1 \cdot 1}{2}\right) \end{aligned}$$

$$= x + \frac{1}{3} \ln|x-1| - \frac{1}{3} \left(\frac{1}{2} \int \frac{2x+1}{x^2+x+1} dx + \frac{3}{2} \int \frac{1}{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}} dx \right)$$

$$= x + \frac{1}{3} \ln|x-1| - \frac{1}{6} \ln|x^2+x+1| - \frac{1}{2} \cdot \frac{4^2}{3} \int \frac{dx}{\frac{4\left(x+\frac{1}{2}\right)^2 + 1}{3}}$$

$$= x + \frac{1}{3} \ln|x-1| - \frac{1}{6} \ln|x^2+x+1| - \underbrace{\frac{\sqrt{3}}{3}}_{1/\sqrt{3}} \operatorname{Arctan}\left(\frac{2x+1}{\sqrt{3}}\right) + C \quad \left\{ \frac{2}{\sqrt{3}} dx = dt \right.$$

ex 8.16(a)

$$t = \tan \frac{\theta}{2} ; \theta = 2 \arctan t ; d\theta = \frac{2}{1+t^2} dt$$

a) $\int \frac{\tan \theta}{1+\sin \theta} d\theta$

$$\sin \theta = \frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$$

$$= \int \frac{2t}{1-t^2} \cdot \frac{1}{1 + \left(\frac{2t}{1+t^2}\right)} \cdot \frac{2}{1+t^2} dt$$

$$\cos \theta = \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}}$$

$$= \int \frac{2t}{1-t^2} \cdot \frac{1+t^2}{1+t^2+2t} \cdot \frac{2}{1+t^2} dt$$

$$= (t+1)^2$$

$$\frac{\sin \theta}{\cos \theta} = \frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} \cdot \frac{1 + \tan^2 \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}}$$

$$= 4 \int \frac{t}{(1-t^2)(t+1)^2} dt$$

$$\Rightarrow \tan \theta = \frac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}}$$

$$= 4 \int \frac{t}{(1-t)(1+t)(t+1)^2} dt$$

$$= 4 \int \frac{t}{(1-t)(t+1)^3} dt$$

geg: $\frac{4t}{(1-t)(t+1)^3} = \frac{1}{2(1-t)} + \frac{1}{2(1+t)} + \frac{1}{(1+t)^2} - \frac{2}{(1+t)^3}$

$$= \int \frac{1}{2(1-t)} dt + \int \frac{1}{2(1+t)} dt + \int \frac{1}{(1+t)^2} dt - \int \frac{2}{(1+t)^3} dt$$

$$= \frac{1}{2} \int \frac{1}{1-t} dt + \frac{1}{2} \int \frac{1}{1+t} dt + \int \frac{1}{(1+t)^2} dt - 2 \int \frac{1}{(1+t)^3} dt$$

$$= -\frac{1}{2} \ln |1-t| + \frac{1}{2} \ln |1+t| - \frac{1}{1+t} - 2 \cdot \frac{1}{-2} \frac{1}{(1+t)^2} + C$$

$$= -\frac{1}{2} \ln |1-t| + \frac{1}{2} \ln |1+t| - \frac{1}{1+t} + \frac{1}{(1+t)^2} + C$$

$$= -\frac{1}{2} \ln \left| \tan \frac{\theta}{2} \right| + \frac{1}{2} \ln \left| 1 + \tan \frac{\theta}{2} \right| - \frac{1}{1 + \tan \frac{\theta}{2}} + \frac{1}{(1 + \tan \frac{\theta}{2})^2} + C$$

$$= \frac{1}{2} \ln \left| \frac{1 + \tan \frac{\theta}{2}}{1 - \tan \frac{\theta}{2}} \right| - \frac{1}{1 + \tan \frac{\theta}{2}} + \frac{1}{(1 + \tan \frac{\theta}{2})^2} + C$$

20/11/2019

oefening 13

oef. 8.24

$$kt = \int_0^x \frac{dx}{(1-x)(3-x)} \quad \text{met } 0 < x < 1$$

$$\int_0^x \frac{dx}{(1-x)(3-x)} \rightarrow \frac{1}{(1-x)(3-x)} = \frac{A}{(1-x)} + \frac{B}{(3-x)}$$

$$1 = 3A - Ax + B - Bx$$

$$\begin{cases} 0 = -A - B \Leftrightarrow A = -B \\ 1 = 3A + B \end{cases} \Leftrightarrow \begin{cases} A = 1/2 \\ B = -1/2 \end{cases}$$

$$= \int_0^x \frac{1/2}{1-x} dx + \int_0^x \frac{-1/2}{3-x} dx = \frac{1}{2} \int_0^x \frac{1}{1-x} dx - \frac{1}{2} \int_0^x \frac{1}{3-x} dx$$

$$1-x=t$$

$$-dx=dt$$

$$x \rightarrow 0, t \rightarrow 1$$

$$x \rightarrow x, t \rightarrow 1-x$$

$$3-x=u$$

$$-dx=du$$

$$x \rightarrow 0, u \rightarrow 3$$

$$x \rightarrow x; u \rightarrow 3-x$$

$$= -\frac{1}{2} \int_1^{1-x} \frac{1}{t} dt + \frac{1}{2} \int_3^{3-x} \frac{1}{u} du = \frac{1}{2} \int_{1-x}^1 \frac{1}{t} dt - \frac{1}{2} \int_{3-x}^3 \frac{1}{u} du$$

$$= -\frac{1}{2} [\ln|1-x|]_0^x + \frac{1}{2} [\ln|3-x|]_0^x$$

terug naar x terug grenzen naar x

$$= -\frac{1}{2} (\ln|1-x| - \ln|1-0|) + \frac{1}{2} (\ln|3-x| - \ln|3-0|)$$

$$= -\frac{1}{2} \ln|1-x| + \frac{1}{2} (\ln|3-x| - \ln 3) = \frac{1}{2} \ln \left| \frac{3-x}{1-x} \right| - \frac{1}{2} \ln 3$$

$$= \frac{1}{2} \ln \left| \frac{3-x}{1-x} \cdot \frac{1}{3} \right| = \frac{1}{2} \ln \left| \frac{3-x}{3-3x} \right|$$

$$kt = \frac{1}{2} \ln \left| \frac{3-x}{3-3x} \right| \Leftrightarrow 2kt = \ln \left| \frac{3-x}{3-3x} \right|$$

$$\Leftrightarrow e^{2kt} = \left| \frac{3-x}{3-3x} \right| \Leftrightarrow e^{2kt} = \frac{3-x}{3-3x} \quad \text{wenn } 0 < x < 1 \quad \text{denn } \frac{3-x}{3-3x} > 0$$

$$\Leftrightarrow e^{2kt} (3-3x) = 3-x \quad \Leftrightarrow 3e^{2kt} - e^{2kt} 3x = 3-x$$

$$\Leftrightarrow 3e^{2kt} - 3 = -x + e^{2kt} 3x \quad \Leftrightarrow 3e^{2kt} - 3 = x(3e^{2kt} - 1)$$

$$\Leftrightarrow \frac{3e^{2kt} - 3}{3e^{2kt} - 1} = -x = \frac{3 - 3e^{2kt}}{1 - 3e^{2kt}}$$

Vorbereitung Wiskunde 1 - oefeningen

oefenzitting 14 - 22/11/2019

oef. 8.17(a)

$$a) \int \frac{dx}{\sqrt{9-x^2}} = \frac{1}{3} \int \frac{dx}{\sqrt{1-\frac{x^2}{9}}} = \int \frac{dt}{\sqrt{1-t^2}} = \arcsin\left(\frac{x}{3}\right) + C$$

$\frac{x}{3} = t$
 $\frac{1}{3} dx = dt$

oef. 8.20(a)

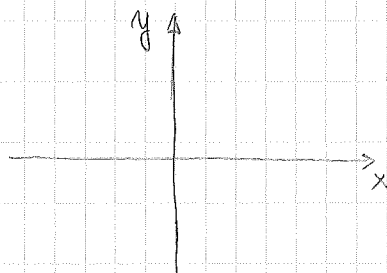
a) $\int_1^e \ln x \, dx$

$\hookrightarrow f(x) = \ln x$
 $g'(x) = 1 \Rightarrow g(x) = x$

$$= [x \ln x]_1^e - \int_1^e \frac{1}{x} \cdot x \, dx = [x \ln x]_1^e - [x]_1^e = e - e + 1 = 1$$

oef. 9.1(c)

c) $f(x, y) = x^2 + 2xy^2$



oef. 9.2(a)

a) $x^2 + 2y^2 + 3z^2 + 4xy + 5xz - 6yz$

$$\frac{\partial f}{\partial x} = 2x + 4y + 5z$$

$$\frac{\partial f}{\partial z} = 6z + 5x - 6y$$

$$\frac{\partial f}{\partial y} = 4y + 4x - 6z$$

Def 9.3 (b)

$$b) \text{ bogen } \frac{y}{x} \text{ in } P(-1, 1) \leadsto \left(\text{bogen } \frac{y}{x}\right)' = \frac{1}{1 + \left(\frac{y}{x}\right)^2} \cdot \left(\frac{y}{x}\right)'$$

$$\frac{\partial f}{\partial x}(-1, 1) \cdot \frac{1}{1 + \left(\frac{1}{-1}\right)^2} \cdot \frac{\overset{1}{y}(-1)}{x^2(-1)} = \frac{-1}{2}$$

$$\frac{\partial f}{\partial y}(-1, 1) \cdot \frac{1}{1 + \left(\frac{1}{-1}\right)^2} \cdot \frac{1}{\frac{x}{y}(-1)} = \frac{-1}{2}$$

22.11.2019

Wiskunde I - oefeningen

oefenzitting 14

oef. 8.11(c)

$$c) \int_0^1 \frac{x^2}{\sqrt{4-3x}} dx$$

$$\hookrightarrow f(x) = x^2$$

$$g'(x) = \frac{1}{\sqrt{4-3x}}$$

$$\Rightarrow g(x) = \frac{+3}{2} \sqrt{4-3x}$$

$$\int f'g' = f \cdot g - \int f'g$$

$$= \left[x^2 \cdot \frac{3}{2} \sqrt{4-3x} \right]_0^1 - \int_0^1 x \cdot \frac{3}{2} \sqrt{4-3x} dx$$

$$= \left[x^2 \cdot \frac{3}{2} \sqrt{4-3x} \right]_0^1 - 3 \left[\int_0^1 x \sqrt{4-3x}^{3/2} dx \right] - \int_0^1 1 \cdot \frac{1}{2} (4-3x)^{3/2} dx$$

$$= \left[x^2 \cdot \frac{3}{2} \sqrt{4-3x} \right]_0^1 - 3 \left[\left[\frac{x}{2} (4-3x)^{3/2} \right]_0^1 - \frac{1}{2} \cdot \frac{1}{2} (4-3x)^{5/2} \cdot \frac{2}{5} \right]_0^1$$

$$= \left(\frac{3}{2} \cdot 1 - 0 \right) - 3 \left[\left(\frac{1}{2} (1) - 0 \right) + \frac{3}{2} \left((1) \frac{2}{5} - 0 \right) \right]$$

$$= \frac{3}{2} - \frac{3}{2} \cdot \frac{10}{10} = \frac{4}{10} = \frac{2}{5}$$

$$\int_0^1 \frac{x^2}{\sqrt{4-3x}} dx \rightarrow u = 4-3x \rightarrow x = \frac{u-4}{-3}$$

$$du = -3dx$$

$$x \rightarrow 1; u \rightarrow 1$$

$$x \rightarrow 0; u \rightarrow 4$$

$$= \int_4^1 \frac{u^2 - 8u + 16}{9\sqrt{u}} \frac{du}{-3} = \frac{-1}{27} \int_4^1 \frac{u^2 - 8u + 16}{\sqrt{u}} du$$

$$= \frac{1}{27} \int_1^4 16 \frac{1}{\sqrt{u}} du - \frac{8}{27} \int_1^4 \frac{u}{\sqrt{u}} du + \frac{1}{27} \int_1^4 \frac{u^2}{\sqrt{u}} du$$

$$= \frac{16}{27} \left[2\sqrt{u} \right]_1^4 - \frac{8}{27} \left[\frac{2}{3} u^{3/2} \right]_1^4 + \frac{1}{27} \left[\frac{2}{5} u^{5/2} \right]_1^4$$

$$= \frac{16}{27} \cdot 2 - \frac{8}{27} \cdot \frac{14}{3} + \frac{1}{27} \cdot \frac{62}{5} = \frac{106}{405}$$

oef. 8.21

Tip:

- zie paragraaf 7.5.2
- denk aan form. v. Pyth.: maak een tekening & doe een subst. v. x naar 1 v. d. hoeken θ

$$p(x) = \sqrt{L^2 - x^2}$$

met $0 \leq x \leq L$

M

* totale massa $\int_0^L \sqrt{L^2 - x^2} dx =$



$$= \int_0^L \sqrt{L^2 - L^2 \sin^2 \theta} \cdot L \cos \theta d\theta$$

$$= \int_0^L L^2 (1 - \sin^2 \theta) \cos \theta d\theta$$

$$x = L \sin \theta$$

$$dx = L \cos \theta d\theta$$

$$x \rightarrow 0; \theta \rightarrow 0$$

$$x \rightarrow L; \theta \rightarrow \arcsin \frac{x}{L}$$

$$= \int_0^L L \cos \theta \cdot L \cos \theta d\theta = \int_0^L L^2 \cos^2 \theta d\theta$$

p. 202!

$$= L^2 \int_0^L \cos^2 \theta d\theta = L^2 \int_0^{\arcsin \frac{x}{L}} \frac{1}{2} (1 + \cos 2\theta) d\theta$$

$$= \frac{L^2}{2} \int_0^{\arcsin \frac{x}{L}} 1 + \cos 2\theta d\theta = \frac{L^2}{2} \left(\int_0^{\arcsin \frac{x}{L}} 1 d\theta + \int_0^{\arcsin \frac{x}{L}} \cos 2\theta d\theta \right)$$

$$= \frac{L^2}{2} \left([\theta]_0^{\arcsin \frac{x}{L}} + \left[\frac{1}{2} \sin 2\theta \right]_0^{\arcsin \frac{x}{L}} \right)$$

$$2\theta = u$$

$$2d\theta = du$$

$$\theta \rightarrow 0; u \rightarrow 0$$

$$\theta \rightarrow \arcsin \frac{x}{L}; u \rightarrow 2 \arcsin \frac{x}{L}$$

$$= \frac{L^2}{2} \left(\arcsin \frac{x}{L} + \frac{1}{2} \sin \left(2 \arcsin \frac{x}{L} \right) \right)$$

$$= \frac{L^2}{2} \left(\arcsin \frac{x}{L} + \frac{1}{2} \sin \frac{2x}{L} \right) = \frac{L^2}{2} \left(\arcsin \frac{x}{L} + \frac{x}{L} \right)$$

oef. 9.1 (e)

Tip: niveaukromme v. f bij c is kromme in xy -vlak waarbij $f(x, y) = c$

e) $f(x, y) = \sqrt{x^2 + y^2}$

oef. 9.6 (a)

Tip: Betekenis: $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial x} \right)$

a) $z = 2x^2y + \cos(x+y)$

$$\frac{\partial^2 z}{\partial x^2} = 4xy + (-\sin(x+y) \cdot (1+y))$$

voorbereiding Wiskunde 1 - oefeningenoefenzitting 15-27/11/2019oef. 9.13(a)

raakvlak

a) $z = x^2 - y^2$ in $P: (-2, 1, 3)$

$$\star \frac{\partial z}{\partial x} = 2x \quad \leadsto \quad 2 \cdot (-2) = -4$$

$$\star \frac{\partial z}{\partial y} = -2y \quad \leadsto \quad -2 \cdot (1) = -2$$

$$\star \text{ raakvlak: } z - 3 = -4(x + 2) - 2(y - 1)$$

oef. 9.16(a)

raaklijn

a) $z = x^2 - y^2$ in $P: (-2, 1)$

$$\star P \text{ invullen: } z = (-2)^2 - 1^2 = 3$$

$$\star \frac{\partial z}{\partial x} = 2x \quad \leadsto \quad 2 \cdot (-2) = -4$$

$$\star \frac{\partial z}{\partial y} = -2y \quad \leadsto \quad -2 \cdot (1) = -2$$

$$\left. \begin{array}{l} \star \frac{\partial z}{\partial x} = 2x \quad \leadsto \quad 2 \cdot (-2) = -4 \\ \star \frac{\partial z}{\partial y} = -2y \quad \leadsto \quad -2 \cdot (1) = -2 \end{array} \right\} \text{ richt} = -\frac{-4}{-2} = -2$$

$$\star \text{ ngl raaklijn: } y = -2x + k$$

$$\hookrightarrow = 1 + 2 \cdot (-2) = -3$$

$$\Leftrightarrow y = -2x - 3$$

24/11/2019

Wiskunde I - oefeningen

oefening 15

oef. 9.13 (c)

c) $z = \ln(1+x^2+y^2)$ in $P: (0,0,0)$

$$\checkmark \frac{\partial z}{\partial x} = \frac{1}{1+x^2+y^2} \cdot 2x \leadsto x=0 \rightarrow 0$$

$$\checkmark \frac{\partial z}{\partial y} = \frac{1}{1+x^2+y^2} \cdot 2y \leadsto y=0 \rightarrow 0$$

• raakvlak: $z = 0$

oef. 9.14

kettingregel (9.4.2) op p. 241

$$z = x^2 + 2xy + 3y^2 \quad \text{met } x = (1+t)^{1/2} \\ y = (1-t)^{1/2}$$

① $\frac{dz}{dt} = \left(\frac{\partial z}{\partial x}\right) \frac{dx}{dt} + \left(\frac{\partial z}{\partial y}\right) \frac{dy}{dt}$

$$\checkmark \frac{dx}{dt} = \frac{1}{2\sqrt{1+t}}$$

$$\checkmark \frac{\partial z}{\partial x} = 2x + 2y$$

$$\checkmark \frac{dy}{dt} = \frac{-1}{2\sqrt{1-t}}$$

$$\checkmark \frac{\partial z}{\partial y} = 2x + 6y$$

② $z = (1+t) + 2 \cdot (1+t)^{1/2} (1-t)^{1/2} + 3(1-t)$

$$z = 4 - 2t + 2(1+t)^{1/2} (1-t)^{1/2}$$

$$\frac{dz}{dt} = -2 + 2 \cdot \frac{1}{2\sqrt{1+t}} \cdot (1-t)^{1/2} + 2 \cdot \frac{1}{2\sqrt{1-t}} \cdot (-1) \cdot (1+t)^{1/2}$$

$$= -2 + \frac{2\sqrt{1-t}}{\sqrt{1+t}} - \frac{2\sqrt{1+t}}{\sqrt{1-t}} = \frac{-2\sqrt{1+t}\sqrt{1-t} + 2\sqrt{1-t}\sqrt{1-t} - 2\sqrt{1+t}\sqrt{1+t}}{\sqrt{1+t}\sqrt{1-t}}$$

$$= -2 + \frac{1-t - (1+t)}{\sqrt{1+t}\sqrt{1-t}} = -2 + \frac{-2t}{\sqrt{1+t}\sqrt{1-t}}$$

neue Regel: Kettenregel

$$x = \sqrt{1+t}$$

$$y = \sqrt{1-t}$$

$$\begin{aligned} \frac{dx}{dt} &= (2x + 2y) \cdot \frac{1}{2\sqrt{1+t}} + (2x + 6y) \cdot \frac{-1}{2\sqrt{1-t}} \\ &= \frac{x+y}{\sqrt{1+t}} - \frac{x+3y}{\sqrt{1-t}} \\ &= \frac{\sqrt{1+t} + \sqrt{1-t}}{\sqrt{1+t}} - \frac{\sqrt{1+t} + 3\sqrt{1-t}}{\sqrt{1-t}} \\ &= \frac{(\sqrt{1+t} + \sqrt{1-t})\sqrt{1-t} - (\sqrt{1+t} + 3\sqrt{1-t})\sqrt{1+t}}{\sqrt{1+t}\sqrt{1-t}} \\ &= \frac{\sqrt{1+t}\sqrt{1-t} + (1-t) - (1+t) - 3\sqrt{1-t}\sqrt{1+t}}{\sqrt{1+t}\sqrt{1-t}} \\ &= \frac{-2t - 2\sqrt{1+t}\sqrt{1-t}}{\sqrt{1+t}\sqrt{1-t}} = -2 - \frac{2t}{\sqrt{1+t}\sqrt{1-t}} \end{aligned}$$

Def. 9.17(a)

a) $2y - z^2 - 3xz = 0$ in $P(1, 5, 2)$

Vergleichung raaktant

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

$$a = \frac{df}{dx} ; b = \frac{df}{dy} ; c = \frac{df}{dz}$$

$$\begin{aligned} \hookrightarrow a &= -3z ; b = 2 ; c = -2z - 3x \hookrightarrow -2(2) - 3(1) = -7 \\ &\hookrightarrow -3(2) \end{aligned}$$

$$\hookrightarrow -6(x-1) + 2(y-5) - 7(z-2) = 0$$

$$-6x + 6 + 2y - 10 - 7z + 14 = 0$$

$$-6x + 2y - 7z + 10 = 0 \Rightarrow -10 = -6x + 2y - 7z$$

oef. 9.22 (b, c)

b) $f(x, y) = \frac{x}{1+y}$ in P. (0, 0) met $\theta = \frac{\pi}{4}$

$$x(t) = x_0 + t \cos \theta ; \quad y(t) = y_0 + t \sin \theta$$

$$\frac{dx}{dt} = \cos \theta ; \quad \frac{dy}{dt} = \sin \theta$$

$$\frac{dz}{dt} \text{ (= richtingsafg. Dof)} = \cos \theta \frac{\partial z}{\partial x} + \sin \theta \frac{\partial z}{\partial y}$$

$$\checkmark \frac{\partial z}{\partial x} = \frac{1}{1+y} \quad \leadsto 1$$

$$\checkmark \frac{\partial z}{\partial y} = \frac{-x}{(1+y)^2} \quad \leadsto 0$$

$$\checkmark D_{\pi/4} f = \cos \frac{\pi}{4} \cdot 1 + \sin \frac{\pi}{4} \cdot 0 = \frac{\sqrt{2}}{2}$$

c) $f(x, y) = x^2 + y^2 + xy$ in P. (1, -1) met θ willekeurig

$$\checkmark \frac{\partial z}{\partial x} = 2x + y \quad \leadsto 2 \cdot 1 - 1 = 1$$

$$\checkmark \frac{\partial z}{\partial y} = 2y + x \quad \leadsto 2 \cdot (-1) + 1 = -1$$

$$\checkmark D_{\theta} f = \cos \theta - \sin \theta$$

oef. 9.23 (b, c)

hoek is max verandering
1) grad f in P berekenen $\left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y} \right)$ in P
↳ vector $\Rightarrow$ we weten de hoek met x-as

2) lengte vector berekenen = richtingsafg
↳ zie w. Pyth

b) $\nabla \text{grad } f(1, 0) \rightarrow \text{hoek} = 0$
 $\nabla \text{ lengte} = 1$

c) $\nabla \text{grad } f(1, -1) \rightarrow \text{hoek} = -\pi/4$
 $\nabla \text{ lengte} = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

oef. 9.24

$$f(x, y) = xy e^{-x^2 - y^2}$$

10.1(a)
10.3(c)

Vorbereitung Wiskunde 1 - oefeningen

oefenzitting 16-29/11/2019

oef. 10.1 (a)

a) $x(t) = e^{2t} - 2$; $x' = 2x + 4$; $x(0) = -2$

$$x' = 2 \underbrace{e^{2t}}_{= (x+2)} = 2(x+2) = 2x + 4 \Rightarrow \text{klopt}$$

$$x(0) = -2 \Rightarrow x(0) = e^0 - 2 = 1 - 2 = -1 \Rightarrow \text{klopt nl}$$

$\Rightarrow$ DV klopt, beginwa niet

oef. 10.3 (c)

c) $x' = (t-1)x \rightarrow \frac{dx}{dt} = (t-1)x$

$$\Leftrightarrow \int \frac{1}{x} dx = \int (t-1) dt$$

$$\Leftrightarrow \ln |x| = \frac{t^2}{2} - t + C$$

$$x = e^{\frac{t^2}{2} - t + C} = \underbrace{e^C}_C \cdot e^{\frac{t^2}{2} - t} \quad (\text{nieuwe } C)$$

$$\int x^m dx = \frac{1}{m+1} x^{m+1} + C$$

oef. 10.3 (d)

$$d) \frac{dy}{dx} = 3x^2 y \Leftrightarrow \int \frac{1}{y} dy = \int 3x^2 dx$$

$$\Leftrightarrow \ln|y| = x^3 + C$$

$$\Leftrightarrow y = e^{x^3 + C} = e^{x^3} \cdot \underset{= e^C}{C}$$

oef. 10.4 (d)

$$d) \frac{dy}{dx} = e^{x+4}; y(0) = 0$$

$$\Leftrightarrow \frac{dy}{dx} = e^x \cdot e^4 \Leftrightarrow \int \frac{1}{e^4} dy = \int e^x dx$$

$$\Leftrightarrow -e^{-4} = e^x + C \Leftrightarrow \frac{-1}{e^4} = e^x + C$$

$$\Leftrightarrow e^4 = \frac{-1}{e^x + C} \Leftrightarrow y = \ln\left(\frac{-1}{e^x + C}\right)$$

$$(y(0) = 0) \Leftrightarrow 0 = \ln\left(\frac{\overset{=1}{-1}}{e^0 + C}\right) \Rightarrow C = -2$$

$$\Leftrightarrow y(x) = \ln\left(\frac{\oplus 1}{2 - e^x}\right)$$

oef 10.5

a) $x' = (x+t+1)^2$

schieden 2 veranderlijken is nt mogelijk omdat bij het RL een functie is x en product is is d functie t

b) $y = x + t + 1$

$$y' = x' + 1$$

$$y' = (x+t+1)^2 + 1$$

$$\frac{dy}{dt} = (x+t+1)^2 + 1 \quad \Leftrightarrow \quad \frac{dy}{dt} = y^2 + 1$$

c) $\Leftrightarrow \frac{1}{y^2+1} dy = dt \quad \Leftrightarrow \text{Bogtan } y = t + C$

$$\Leftrightarrow \tan(\text{Bogtan } y) = \tan(t+C)$$

$$\Leftrightarrow y = \tan(t+C)$$

$$\Leftrightarrow x+t+1 = \tan(t+C)$$

$$\Leftrightarrow x = -t-1 + \tan(t+C)$$

oef 10.7(a,b)

$$a'(t) = -2\pi [a(t)]^2 \quad (\pi > 0)$$

a) $\frac{da}{dt} = -2\pi (a(t))^2$

$$\Leftrightarrow \int \frac{1}{a^2} da = -2\pi \int dt \quad \Leftrightarrow \frac{-1}{a} = -2\pi t + C$$

$$\Leftrightarrow a(t) = \frac{1}{2\pi t - C}$$

b) $\lim_{t \rightarrow +\infty} a(t) = \lim_{t \rightarrow +\infty} \frac{1}{2\pi t - C} = 0$

$a(t) = \frac{1}{2\pi t - C}$
b) 0

Def. 10.9 (g)

$$g) \quad y' = \frac{4y}{x(y-3)} \quad \Leftrightarrow \quad \frac{dy}{dx} = \frac{4y}{x(y-3)}$$

$$\Leftrightarrow \quad \frac{1}{4y} dy = \frac{1}{x(y-3)} dx \quad \Leftrightarrow \quad \frac{y-3}{4y} dy = \frac{1}{x} dx$$

$$\Leftrightarrow \quad \frac{1}{4} y - \frac{3}{4} \ln |y| = \ln |x| + C$$

$$\Leftrightarrow \quad y - 3 \ln |y| = 4 \ln |x| + C$$

4/12/2019

Wiskunde I - oefeningen

oefening 17

oef 10 11 (a, b, c)

$$a'(t) = -\pi a(t) b(t)$$

$$b'(t) = -\pi a(t) b(t)$$

met $\pi > 0$

a) $\frac{da}{dt} = -\pi a(t) b(t)$

$$\frac{db}{dt} = -\pi a(t) b(t)$$

$$\Rightarrow b'(t) - a'(t) = 0$$

$$\Rightarrow b(t) - a(t) = cte$$

$$\Leftrightarrow b(t) = a(t) + c$$

b) $a'(t) = -\pi a(t) (a(t) + c)$

$$\frac{da}{dt} = -\pi a(t) (a(t) + c)$$

* als $c = 0$:

$$\frac{da}{dt} = -\pi a^2 \Leftrightarrow \frac{1}{a^2} da = -\pi dt$$

$$\Leftrightarrow -\frac{1}{a} = -\pi t + K \Leftrightarrow a(t) = \frac{1}{\pi t + K} \quad \text{en } b(t) = \frac{1}{\pi t + K} + \underbrace{c}_0$$

* als $c \neq 0$

$$\frac{da}{dt} = -\pi a^2 - \pi ac = -\pi (a^2 + ac)$$

$$\Leftrightarrow \frac{1}{a^2 + ac} da = -\pi dt = \frac{1}{a(a+c)} da$$

$$\hookrightarrow \frac{1}{a(a+c)} = \frac{A}{a} + \frac{B}{a+c}$$

$$1 = A(a+c) + Ba$$

$$\begin{cases} 0 = A+B \\ 1 = +Ac \end{cases} \Leftrightarrow \begin{cases} B = -1/c \\ A = \frac{1}{c} \end{cases}$$

$$\Leftrightarrow \frac{1}{c} \int \frac{1}{a} da - \frac{1}{c} \int \frac{1}{a+c} da = -rt + K$$

$$\Leftrightarrow \frac{1}{c} \ln|a| - \frac{1}{c} \int \frac{1}{a+c} da = -rt + K$$

$$\Leftrightarrow \frac{1}{c} \ln|a| - \frac{1}{c} \ln|a+c| = -rt + K$$

$$\Leftrightarrow \frac{1}{c} (\ln|a| - \ln|a+c|) = -rt + K$$

$$\Leftrightarrow \frac{1}{c} \ln \left| \frac{a}{a+c} \right| = -rt + K$$

$$\Leftrightarrow \ln \left(\frac{a+c}{a} \right) = +rt + K$$

$$\Leftrightarrow \frac{a+c}{a} = e^{rt+c} \cdot \overset{e^K = cte}{K}$$

$$\Leftrightarrow a+c = a \cdot e^{rt+c} K$$

$$\Leftrightarrow a = \frac{c}{Ke^{rt}-1} \quad \text{en } b = \frac{c}{Ke^{rt}-1} + c$$

$$\Leftrightarrow \frac{c+c(Ke^{rt}-1)}{Ke^{rt}-1} = \frac{c}{1-Ke^{-rt}}$$

c) * c = 0:

$$\lim_{t \rightarrow +\infty} a(t) = \lim_{t \rightarrow +\infty} \frac{1}{rt+K} = 0$$

$$\lim_{t \rightarrow +\infty} b(t) = \lim_{t \rightarrow +\infty} \frac{1}{rt+K} = 0$$

* c < 0:

$$\lim_{t \rightarrow +\infty} a(t) = \lim_{t \rightarrow +\infty} \frac{c}{\underbrace{Ke^{rt}-1}_{=0}} = \frac{c}{-1} = -c$$

$$\lim_{t \rightarrow +\infty} b(t) = \lim_{t \rightarrow +\infty} \frac{c}{\underbrace{1-Ke^{-rt}}_{=1}} = 0$$

$$* \underline{C > 0}$$

$$\lim_{t \rightarrow +\infty} a(t) = \lim_{t \rightarrow +\infty} \frac{C}{ke^{4tc} - 1} = 0$$

$$\lim_{t \rightarrow +\infty} b(t) = C$$

oef. 10.14 (b)

- enkel termen met x
- 1) mbr. homogene DV
 - 2) variatie n. de
- ↳ particuliere, dus met andere factoren

$$b) \quad x' = 4x + 2t$$

$$\Leftrightarrow x'(t) = 4x(t) + 2t$$

$$\Leftrightarrow x'(t) - 4x(t) - 2t = 0 \quad (*)$$

↳ ① homogeen : $x'(t) - 4x(t) = 0$

$$\Leftrightarrow \frac{dx}{dt} = 4x$$

$$\Leftrightarrow \frac{1}{x} dx = 4 dt$$

$$\Leftrightarrow \ln|x| = 4t + C$$

$$\Leftrightarrow x = e^{4t+C}$$

$$\Leftrightarrow x = C e^{4t} \quad (x(t) = C \cdot e^{4t})$$

↳ ② Particulier : $x(t) = \underline{y(t)} \cdot e^{4t}$

↳ cte laten variëren

↳ afleiden :

$$x'(t) = \underbrace{y(t) e^{4t}}_{x(t)} 4 + y'(t) e^{4t}$$

oorspr. DV gebruiken (*) om te vereenvoudigen x y(t) te vinden

$$\Leftrightarrow x'(t) - 4x(t) = y'(t) e^{4t}$$

$$\Leftrightarrow 2t = y'(t) e^{4t}$$

$$\Leftrightarrow \frac{2t}{e^{4t}} = \frac{dy}{dt}$$

$$\Leftrightarrow \int \frac{2t}{e^{4t}} dt = \int dy$$

$$\Leftrightarrow \int \frac{2t}{e^{4t}} dt = y$$

→ parti intégrale: $\int \frac{2t}{e^{4t}} dt = 2 \int t \cdot e^{-4t}$

$\hookrightarrow f(t) = t$
 $g'(t) = e^{-4t} \Rightarrow g(t) = \frac{e^{-4t}}{-4}$

$$= 2 \left[t \cdot \frac{e^{-4t}}{-4} - \int 1 \cdot \frac{e^{-4t}}{-4} dt \right]$$

$$= \frac{-t}{2} e^{-4t} + 2 \cdot \frac{1}{4} \cdot \frac{-1}{4} \cdot e^{-4t} = \frac{-t}{2} e^{-4t} - \frac{1}{8} e^{-4t} + C$$

$$\Rightarrow N_f = \frac{-t}{2} e^{-4t} - \frac{1}{8} e^{-4t} + C$$

Dus $x(t) = N_f(t) e^{4t}$

$$= \left(\frac{-t}{2} e^{-4t} - \frac{1}{8} e^{-4t} + C \right) e^{4t}$$

$$= \frac{-t}{2} - \frac{1}{8} + C \cdot e^{4t} \quad (\text{alg. op.})$$

Oef. 10.17

$$T'(t) = k (T_{\text{omg}} - T(t)) \quad (k > 0)$$

a) $T(0) = -20^\circ\text{C}$ en $T(20) = 20^\circ\text{C}$

$T_{\text{omg}} = 200^\circ\text{C}$

$k = 0,01003$

~~$\Rightarrow T(t) = 2t - 20$ dus $T'(t) = 2$~~

~~invullen: $2 = k(200^\circ\text{C})$~~

$200 - T(t) = u$
 $-1 dT = du$

$$T'(t) = k(200 - T(t))$$

$$\Leftrightarrow \int \frac{1}{200 - T(t)} dT = \int k dt$$

$$\Leftrightarrow T'(t) = 200k - kT(t)$$

$$\Leftrightarrow -\ln|200 - T| = kt$$

$$\Leftrightarrow \frac{dT}{dt} = 200k - kT(t)$$

$$\Leftrightarrow \frac{-\ln|200 - T|}{t} = k$$

$$\Leftrightarrow kT(t) dt = 200k dt$$

$$\Leftrightarrow k \frac{T^2(t)}{2} = 200kt + C$$

$$\Leftrightarrow k$$

6/12/2019

Wiskunde I - oefeningen

oefenzitting 18

oef 11.3(c)

$$c) \frac{d^2 x}{dt^2} - 4 \frac{dx}{dt} + 13x = 0$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

Lineaire DV v. 2^e orde:

$$\underbrace{\frac{d^2 x(t)}{dt^2} + a \frac{dx(t)}{dt} + b x(t)}_{\text{homogeen deel}} = \underbrace{\pi(t)}_{\text{inhomogeen deel}} \quad (*)$$

STAP 1: homogene opl $x_h(t)$ vinden:

$$\frac{d^2 x(t)}{dt^2} + a \frac{dx(t)}{dt} + b x(t) = 0$$

STAP 2: particuliere opl $x_p(t)$ vinden (hangt af v. $\pi(t)$)

↳ zie tabel p 285 en tekst p 286

STAP 3: opl v. DV (*)

$$x(t) = x_h(t) + x_p(t)$$

STAP 4: 2 constanten, C_1 en C_2 , in opl $x(t)$ bepalen met beginw.

oef 11.3(c)

$$x(t) = e^{\lambda t} \quad ; \quad \frac{dx}{dt} = \lambda e^{\lambda t} \quad ; \quad \frac{d^2 x}{dt^2} = \lambda^2 e^{\lambda t}$$

$$p. 278-279 \Rightarrow \lambda^2 e^{\lambda t} - 4\lambda e^{\lambda t} + 13 e^{\lambda t} = 0$$

$$\Leftrightarrow e^{\lambda t} (\lambda^2 - 4\lambda + 13) = 0 \rightarrow \frac{1}{2} (-a + \sqrt{a^2 - 4b})$$

$$\Rightarrow \lambda_1 = \frac{1}{2} (4 + \sqrt{(-4)^2 - 4 \cdot 13}) \Rightarrow D < 0$$

$$\lambda_2 = \frac{1}{2} (4 - \sqrt{(-4)^2 - 4 \cdot 13}) \Rightarrow D < 0$$

$$\text{omdat } \Delta < 0 \quad a^2 - 4b \Rightarrow \pm \sqrt{\Delta} = \pm i\sqrt{-\Delta} = \pm i\sqrt{4b - a^2}$$

$$\lambda_1 = (4 + i\sqrt{36}) \frac{1}{2} = 2 + 3i$$

$$\lambda_2 = 2 - 3i$$

$$\Rightarrow x_1(t) = e^{(2+3i)t} = e^{2t+3it}$$

$$x_2(t) = e^{(2-3i)t} = e^{2t-3it}$$

$$\Leftrightarrow x_1(t) = e^{2t} (\cos 3t + i \sin 3t)$$

$$x_2(t) = e^{2t} (\cos 3t - i \sin 3t)$$

$$\Rightarrow x(t) = C_1 e^{2t} \cos 3t + C_2 e^{2t} \sin 3t$$

oef 11.4 (d)

$$d) \frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 9y = 0 ; y(0) = 1, \frac{dy}{dx}(0) = 0$$

$$\Rightarrow \lambda^2 - 6\lambda + 9 = (\lambda - 3)^2$$

met $\lambda = 3$ als dubbel nulpunt

$$\Rightarrow (\text{alg opl}) : y(x) = C_1 e^{3x} + C_2 t e^{3x}$$

$$\Rightarrow y'(x) = 3C_1 e^{3x} + 3C_2 e^{3x} + C_2 e^{3x}$$

$$\Rightarrow y(0) = C_1 \quad \text{en} \quad y'(0) = 3C_1 + C_2$$

om aan beginwaarde voldaan als:

$$1 = C_1 \quad \text{en} \quad 0 = 3C_1 + C_2$$

$$\Leftrightarrow C_1 = 1 \quad \text{en} \quad C_2 = -3$$

Dus opl. v. n. beginwaarde probl. is:

$$y(x) = e^{3x} - 3x e^{3x}$$

Oef 11.5 (c)

$$c) \frac{d^2 x}{dt^2} - 3 \frac{dx}{dt} - 4x = 3te^{2t}$$

$$\star x_h(t) = \frac{d^2 x}{dt^2} - 3 \frac{dx}{dt} - 4x = 0$$

$$x(t) = e^{\lambda t} ; \frac{dx}{dt} = \lambda e^{\lambda t} ; \frac{d^2 x}{dt^2} = \lambda^2 e^{\lambda t}$$

$$\Rightarrow \lambda^2 e^{\lambda t} - 3\lambda e^{\lambda t} - 4e^{\lambda t} = 0$$

$$\Leftrightarrow e^{\lambda t} (\lambda^2 - 3\lambda - 4) = 0$$

$$\Rightarrow \lambda_1 = \frac{1}{2} (3 + \sqrt{3^2 - 4 \cdot (-4)}) = \frac{8}{2} = 4$$

$$\lambda_2 = \frac{3 - 8}{2} = -1$$

$$\Rightarrow x_1(t) = e^{4t}$$

$$x_2(t) = e^{-t}$$

$$\Rightarrow x_h(t) = c_1 e^{4t} + c_2 e^{-t}$$

$$\star p(t) = 3te^{2t}$$

en volgens tabel $(A_0 + A_1 t) e^{2t} = x(t)$

$$\Rightarrow (A_0 + A_1 t) e^{2t} = x(t)$$

en $x'(t) = A_0 \cdot 2e^{2t} + A_1 e^{2t} + A_1 t \cdot 2e^{2t}$

$$x''(t) = 4A_0 e^{2t} + 2A_1 e^{2t} + A_1 \cdot 2e^{2t} + A_1 t \cdot 4e^{2t}$$

$$\Rightarrow x''(t) - 3x'(t) - 4x(t)$$

$$= (4A_0 e^{2t} + 2A_1 e^{2t} + A_1 \cdot 2e^{2t} + A_1 t \cdot 4e^{2t}) - 3(A_0 \cdot 2e^{2t} + A_1 e^{2t} + A_1 t \cdot 2e^{2t}) - 4(A_0 e^{2t} + A_1 t e^{2t})$$

$$= e^{2t} (4A_0 + 2A_1 + 2A_1 + 4A_1 t - 6A_0 - 3A_1 - 6A_1 t - 4A_0 - 4A_1 t)$$

$$= e^{2t} (-6A_0 + A_1 - 6A_1 t) =$$

$$\Rightarrow e^{2t} \cdot \dots = 3te^{2t}$$

Waarom?

resultaat: $A_0 = -1/12$
 $A_1 = -1/2$

$$x_p(t) = \left(-\frac{1}{12} - \frac{1}{2} \right) e^{2t}$$

$$\begin{aligned} * x(t) &= x_h(t) + x_p(t) \\ &= C_1 e^{4t} + C_2 e^{-t} + \left(-\frac{1}{12} - \frac{1}{2} \right) e^{2t} \end{aligned}$$

oef 11.6

$$\frac{d^2 y}{dx^2} + \frac{dy}{dx} - 2y = 0 \quad ; \quad y(0) = 2 \quad \text{en} \quad y(x) \rightarrow 0 \quad \text{als} \quad x \rightarrow \infty$$

$$\Rightarrow \lambda^2 + \lambda - 2 = (\lambda - 1)(\lambda + 2)$$

↳ nulptn $\lambda_1 = 1$
 $\lambda_2 = -2$

$$\text{dus } y(x) = C_1 e^x + C_2 e^{-2x}$$

$$\leadsto y(0) = C_1 + C_2 = 2 \Leftrightarrow C_1 = 2 - C_2$$

$$\leadsto \lim_{x \rightarrow \infty} (C_1 e^x + C_2 e^{-2x}) = \lim_{x \rightarrow \infty} ((2 - C_2) e^x + C_2 e^{-2x})$$

$$= \underbrace{\lim_{x \rightarrow \infty} ((2 - C_2) e^x)}_{\text{gaat naar } \infty} + \underbrace{\lim_{x \rightarrow \infty} (C_2 e^{-2x})}_{= 0}$$

↳ gaat naar ∞ , behalve wanneer $C_2 = 2 \rightarrow \lim = 0$
 dus $C_2 = 2$
 $C_1 = 0$

$$\Rightarrow y(x) = 2e^{-2x}$$

Def 11.7 (c)

$$c) \frac{d^2 x}{dt^2} + 9x = 0 \quad ; \quad x\left(\frac{\pi}{3}\right) = 0 \quad ; \quad \frac{dx}{dt}\left(\frac{\pi}{3}\right) = -1$$

$$\Rightarrow \lambda^2 + 9$$

$$\hookrightarrow \lambda = \pm 3i$$

$$\leadsto x(t) = c_1 \cos 3t + c_2 \sin 3t \quad ; \quad x(\pi/3) = -c_1 = 0$$

$$x'(t) = -3c_1 \sin 3t + \underset{c_2}{3} \cos 3t \quad ; \quad x'(\pi/3) = -3c_2 = -1$$

$$\Rightarrow c_2 = \frac{1}{3}$$

$$\Rightarrow x(t) = \frac{1}{3} \sin 3t$$

Def 11.10

oef 11.12 (c) $\rightarrow$ zoek de homogene opl.

c) $\frac{d^2 x}{dt^2} + x = \cos t$

* homogene opl: $x_h(t)$

$$\frac{d^2 x}{dt^2} + x = 0$$

$$\Rightarrow \lambda^2 + 1 = \pm i$$

$$x_h(t) = \cancel{e^{it} + e^{-it}} = C_1 \cos t + C_2 \sin t$$

oef 11.13 (a, b) $\rightarrow$ zoek de homogene opl

a) $\frac{d^2 x}{dt^2} + 2 \frac{dx}{dt} + x = e^{-t}$; $x(0) = 1$; $\frac{dx}{dt}(0) = 0$

* homogene opl: $x_h(t)$

$$\frac{d^2 x}{dt^2} + 2 \frac{dx}{dt} + x = 0$$

$$\Rightarrow \lambda^2 + 2\lambda + 1 = (\lambda + 1)^2$$

 $\hookrightarrow$ nulpunt $\lambda = -1$ (2v)

$$x(t) = C_1 e^{-t} + C_2 t e^{-t} ; x(0) = C_1 = 1$$

$$x'(t) = -C_1 e^{-t} + C_2 e^{-t} - C_2 t e^{-t} ; x'(0) = -C_1 + C_2 = 0$$

$$\Rightarrow C_2 = 1$$

$$\text{Dus: } x_h(t) = e^{-t} + t e^{-t}$$

$$b) \frac{d^2 x}{dt^2} + 2 \frac{dx}{dt} + \underline{t} = e^{-t} ; x(0) = 1 ; x'(0) = 0$$

$$\Rightarrow x'' + 2x' = e^{-t} - t$$

* homogeneous op : $x_H(t)$

~~$$\frac{d^2 x}{dt^2} + 2 \frac{dx}{dt} + t = 0$$

$$\Rightarrow e^{\lambda t} (\lambda^2 + 2\lambda) + t = 0$$~~

$$\frac{d^2 x}{dt^2} + 2 \frac{dx}{dt} = 0$$

$$\lambda^2 + 2\lambda = 0$$

$$\hookrightarrow \lambda(\lambda + 2) = 0$$

$\hookrightarrow$ nulptr : $\lambda = 0$ en $\lambda = -2$

$$x_{H_1}(t) = e^{0t}$$

$$x_{H_1}' = 1$$

$$\text{en } x_{H_2}(t) = e^{-2t}$$

$$\text{en } x_{H_2}'(t) = -2e^{-2t}$$

$$\Rightarrow \boxed{\begin{aligned} x_H(t) &= C_1 \cdot 1 + C_2 \cdot e^{-2t} \\ x_H'(t) &= -C_2 \cdot 2e^{-2t} \end{aligned}} \rightarrow \begin{aligned} x(0) &= C_1 + C_2 = 1 \\ x'(0) &= -2C_2 = 0 \Rightarrow C_2 = 0 \end{aligned}$$

$$\Leftrightarrow C_1 = 1$$

11/12/2019

Wiskunde 1 - oefeningenoefenzitting 19oef 11.12 (c)

c) * $x_H(t) = C_1 \cos t + C_2 \sin t$

$x'' + x = \cos t$

* $x_p(t) = ?$

$x(t) = At \cos t + Bt \sin t$

$x'(t) = A \cos t + At(-\sin t) + B \sin t + Bt \cos t$

$x''(t) = -A \sin t + A(-\sin t) + At(-\cos t) + B(\cos t) + B \cos t + Bt(-\sin t)$

$\Rightarrow x''(t) + x(t)$

$= (-A \sin t - A \sin t - At \cos t + B \cos t + B \cos t - Bt \sin t) + (At \cos t + Bt \sin t)$

$= -2A \sin t + 2B \cos t = \cos t$

$\Leftrightarrow A = 0 \text{ en } B = \frac{1}{2}$

$\Rightarrow x_p(t) = \frac{1}{2} t \sin t$

* $x(t) = x_p(t) + x_H(t) = \frac{1}{2} t \sin t + C_1 \cos t + C_2 \sin t$

oef 11.13 (a, b)

a) * $x_H = e^{-t} + te^{-t}$

$x'' + 2x' + x = e^{-t}$

* $x_p = ?$

$x(t) = Ae^{-t}$

$x'(t) = -Ae^{-t}$

$x''(t) = Ae^{-t}$

$\Rightarrow x''(t) + 2x'(t) + x(t)$

$= Ae^{-t} + 2(-Ae^{-t}) + Ae^{-t}$

$= 0$

$\Rightarrow x(t) = Ate^{-t}$

$x'(t) = Ae^{-t} + (-Ate^{-t})$

$x''(t) = -Ae^{-t} + (-Ae^{-t}) + (Ate^{-t})$

$\Rightarrow = -Ae^{-t} - Ae^{-t} + Ate^{-t}$

$+ 2(Ae^{-t} - Ate^{-t}) + Ate^{-t}$

$= 0$

$$\Rightarrow x(t) = A t^2 e^{-t}$$

$$x'(t) = 2A t e^{-t} + (-A t^2 e^{-t})$$

$$x''(t) = 2A e^{-t} - 2A t e^{-t} - 2A t e^{-t} + A t^2 e^{-t}$$

$$\Rightarrow 2A e^{-t} - \cancel{2A t e^{-t}} - \cancel{2A t e^{-t}} + A t^2 e^{-t} + \cancel{2(-2A t e^{-t} - A t^2 e^{-t})}$$

$$= 2A e^{-t} = e^{-t}$$

$$\Leftrightarrow 2A = 1 \quad \Leftrightarrow A = \frac{1}{2}$$

$$\Rightarrow x_p(t) = \frac{1}{2} t^2 e^{-t}$$

$$\star \text{ alg. op. } x(t) = \frac{1}{2} t^2 e^{-t} + e^t + t e^{-t}$$

$$b) \star x_H(t) = C_1 + C_2 e^{-2t}$$

$$x'' + 2x' = e^{-t} - t$$

$$\star x_p(t) = ?$$

$$x(t) = A e^{-t} + B t^2 + C t$$

$$x'(t) = -A e^{-t} + 2B t + C$$

$$x''(t) = A e^{-t} + 2B$$

→ enkel dit stuk v. veelterm
x t

$$\Rightarrow A e^{-t} + 2B + 2(-A e^{-t} + 2B t + C)$$

$$= -A e^{-t} + 4B t + 2(B + C) = e^{-t} - t$$

$$\Rightarrow \begin{cases} -A = 1 \\ 4B = -1 \\ 2(B + C) = 0 \end{cases} \Leftrightarrow \begin{cases} A = -1 \\ B = -1/4 \\ C = 1/4 \end{cases}$$

$$\Rightarrow x_p(t) = -e^{-t} - \frac{1}{4} t^2 + \frac{1}{4} t$$

$$\star x(t) = C_1 + C_2 e^{-2t} - e^{-t} - \frac{1}{4} t^2 + \frac{1}{4} t$$

$$\hookrightarrow x(0) = C_1 + C_2 - 1 = 1$$

$$\Leftrightarrow C_1 + C_2 = 2 \Rightarrow C_1 = \frac{11}{8}$$

$$x'(t) = -2C_2 e^{-2t} + e^t - \frac{1}{2} t + \frac{1}{4}$$

$$\hookrightarrow x'(0) = -2C_2 + 1 + \frac{1}{4} = 0 \Leftrightarrow 2C_2 = \frac{5}{4} \Rightarrow C_2 = \frac{5}{8}$$

$$\Rightarrow x(t) = \frac{11}{8} + \frac{5}{8} e^{-2t} - e^{-t} - \frac{1}{4} t^2 + \frac{1}{4} t$$

oef. 11.14

$$u''(x) = -\omega^2 u(x) \quad ; \quad u(0) = 0 \quad ; \quad u(1) = 0$$

$$\Leftrightarrow u''(x) + \omega^2 u(x) = 0$$

$$e^{\lambda x} (\lambda^2 + \omega^2)$$

$$\lambda^2 + \omega^2 = 0$$

$$\hookrightarrow \text{multipen: } \lambda = \pm i\omega$$

$$u_H(x) = c_1 \cos(\omega x) + c_2 \sin \omega x$$

$$\ast u(0) = 0$$

$$= c_1 \cdot \cos 0 + c_2 \sin 0$$

$$= c_1 \cdot 1 + c_2 \cdot 0 = c_1 = 0$$

$$\ast u(1) = 0$$

$$= c_1 \cdot \cos \omega + c_2 \sin \omega$$

$$= 0 \cdot \cos \omega + c_2 \sin \omega$$

$$= c_2 \sin \omega = 0$$

$$\Rightarrow \omega = k\pi \text{ met } k \in \mathbb{Z}$$

oef. 11.18

$$t^2 \frac{d^2 x}{dt^2} + At \frac{dx}{dt} + Bx = 0 \quad \text{voor } t > 0$$

$$y(\lambda) = x(e^\lambda) \rightarrow y''(\lambda) + (A-1)y'(\lambda) + By = 0$$

$$\cancel{y'(\lambda) = e^\lambda + \lambda e^\lambda}$$

$$\cancel{y''(\lambda) = e^\lambda + e^\lambda + \lambda e^\lambda = 2e^\lambda + \lambda e^\lambda}$$

$$\Rightarrow \cancel{2e^\lambda + \lambda e^\lambda + (A-1)(e^\lambda + \lambda e^\lambda) + B(\lambda e^\lambda) = 0}$$

$$\Leftrightarrow \cancel{e^\lambda (2 + \lambda + (A-1) + (A-1)\lambda + B\lambda) = 0}$$

oef. 12.1 (a) → kijk terug nr. oef. 2.21

$$a) f(x, y) = 2x^2 - y^2$$

oef. 12.2 (b) → zoek de stationaire ptn

$$b) x^3 + y^2 - 3x - 4y + 2$$

$$\frac{\partial f}{\partial x} = 3x^2 - 3$$

$$\frac{\partial f}{\partial y} = 2y - 4$$

$$\left. \begin{array}{l} \frac{\partial f}{\partial x} = 3x^2 - 3 \\ \frac{\partial f}{\partial y} = 2y - 4 \end{array} \right\} \Rightarrow \begin{cases} 3x^2 - 3 = 0 \\ 2y - 4 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x^2 - 1 = 0 \\ y - 2 = 0 \end{cases} \Leftrightarrow \begin{cases} x = \pm 1 \\ y = 2 \end{cases}$$

$\Rightarrow$ 2 stat. ptn: $(-1, 2)$ en $(1, 2)$

oef. 12.4 (f) → bereken de partiële afgeleiden 1^o en 2^o orde

$$f) (x^2 + y^2) e^{-x-y}$$

$$\frac{\partial f}{\partial x} = 2x e^{-x-y} + (x^2 + y^2) \cdot (-e^{-x-y})$$

$$\frac{\partial f}{\partial y} = 2y e^{-x-y} + (x^2 + y^2) \cdot (-e^{-x-y})$$

13/12/2019

Wiskunde 1 - oefeningen

oefenzitting 20

oef. 12.2 (b)

nodig oef.: Eig. 12.2.3 p. 300

8te 12.4.1 p. 303

stat. ptn = $(-1, 2)$ en $(1, 2)$

vgl. (12.5.7) p. 309

* (p. 303)

$$\frac{\partial^2 f}{\partial x^2} = 6x$$

$$\frac{\partial^2 f}{\partial x \partial y} = 0$$

$$\frac{\partial^2 f}{\partial y^2} = 2$$

$$\Delta = (6x)(2) - 0^2 = 8x \leadsto x = -1$$

$$\hookrightarrow 8 \cdot (-1) = -8$$

$$\Rightarrow (-1, 2) = \text{zadelpnt}$$

$$\leadsto x = 1$$

$$\hookrightarrow 8 \cdot 1 = 8$$

$$\Rightarrow (1, 2) = \text{lok. min}$$

oef. 12.4 (f)

$$f) \quad \frac{\partial f}{\partial x} = 2xe^{-x-y} + (x^2 + y^2)(-e^{-x-y})$$

$$\frac{\partial f}{\partial y} = 2ye^{-x-y} + (x^2 + y^2)(-e^{-y-x})$$

$$\frac{\partial^2 f}{\partial x^2} = 2e^{-x-y} + 2x(-e^{-x-y}) + (2x)(-e^{-x-y}) + (x^2 + y^2)(e^{-x-y})$$

$$= 2e^{-x-y} + 4x(-e^{-x-y}) + (x^2 + y^2)(e^{-x-y})$$

$$\frac{\partial^2 f}{\partial y^2} = 2e^{-x-y} + 2y(-e^{-y-x}) + 2y(-e^{-y-x}) + (x^2 + y^2)(e^{-y-x})$$

$$= 2e^{-x-y} + 4y(-e^{-y-x}) + (x^2 + y^2)(e^{-y-x})$$

$$\frac{\partial^2 f}{\partial x \partial y} = -2ye^{-x-y} + (2x)(-e^{-y-x}) + (x^2 + y^2)(e^{-x-y})$$

$$\rightarrow \text{stat. ptn: } \begin{cases} 2xe^{-x-y} + (x^2+y^2)(-e^{-x-y}) = 0 \\ 2ye^{-x-y} + (x^2+y^2)(-e^{-x-y}) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} 2xe^{-x-y} = (x^2+y^2)(e^{-x-y}) \\ 2ye^{-x-y} = (x^2+y^2)(e^{-x-y}) \end{cases} \quad \left. \vphantom{\begin{cases} 2xe^{-x-y} = (x^2+y^2)(e^{-x-y}) \\ 2ye^{-x-y} = (x^2+y^2)(e^{-x-y}) \end{cases}} \right\} x=y$$

$$\Leftrightarrow 2x \cancel{e^{-x-x}} = (x^2+x^2) \cancel{(e^{-x-x})}$$

$$\Leftrightarrow 2x = (2x^2) \Leftrightarrow x = x^2$$

$$\hookrightarrow x = 1 \text{ or } x = 0 \\ = y \quad = y$$

$$\Rightarrow \cancel{(1,0)}; (1,1); \cancel{(0,1)}; (0,0)$$

$$\rightarrow \Delta = (2e^{-x-y} + 4x(-e^{-x-y}) + (x^2+y^2)(e^{-x-y})) \\ \cdot (2e^{-x-y} + 4y(-e^{-x-y}) + (x^2+y^2)(e^{-x-y})) \\ - (-2ye^{-x-y} + 2x(-e^{-x-y}) + (x^2+y^2)(e^{-x-y}))^2$$

$$\Rightarrow \Delta(0,0) = (2+0+0) \cdot (2+0+0) - \underbrace{(-2 \cdot 0 + 0 + 0)}_{=0}^2$$

$$\hookrightarrow (0,0) = \text{lok. min}$$

$$\Rightarrow \Delta(1,1) = (2e^{-2} - 4(e^{-2}) + (2)(e^{-2})) \\ \cdot (2e^{-2} - 4(e^{-2}) + (2)(e^{-2})) \\ - (-2e^{-2} - 2(e^{-2}) + 2(e^{-2})^2) \cdot \overset{x^{-2}}{e^{-2}} \overset{e^{-2}}{e^{-2}} \overset{e^{-2}}{e^{-2}} \\ = e^{-2} ((2-4+2)(2-4+2) - (-2-2+2)) \\ = e^{-2} - (-2e^{-2})^2 = e^{-2} - 4e^{-4} < 0$$

$$\hookrightarrow (1,1) = \text{saddelpnt}$$

oef. 12.5 (b)

b) $\cos(x+y)$

$$\left. \begin{aligned} \frac{\partial f}{\partial x} &= -\sin(x+y) \cdot 1 \\ \frac{\partial f}{\partial y} &= -\sin(x+y) \cdot 1 \end{aligned} \right\} \rightarrow \begin{cases} -\sin(x+y) = 0 \\ -\sin(x+y) = 0 \end{cases} \Leftrightarrow \begin{cases} \sin(x+y) \\ \sin(x+y) \end{cases} = 0$$

$$\Rightarrow x+y = k\pi \text{ met } k \in \mathbb{Z}$$

$$\left. \begin{aligned} \frac{\partial^2 f}{\partial x^2} &= -\cos(x+y) \\ \frac{\partial^2 f}{\partial y^2} &= -\cos(x+y) \end{aligned} \right\} \frac{\partial^2 f}{\partial x \partial y} = \sin(x+y)$$

$$\rightarrow \Delta = \cos^2(k\pi) - \sin^2(k\pi) = 0$$

$$\Rightarrow x+y = k\pi \text{ (met } k \in \mathbb{Z}), \text{ dan } \cos(x+y) = -1 \text{ als } k \text{ is oneven} \rightarrow \min$$

$$\cos(x+y) = 1 \text{ als } k \text{ is even} \rightarrow \max$$

oef. 12.7 (b, c)

b) $f(x, y) = x^3 y + 3xy^3 - 3xy$

$\rightarrow$ stat. pnt?

$$\hookrightarrow \frac{\partial f}{\partial x} = 3x^2 y + 3y^3 - 3y$$

$$\frac{\partial f}{\partial y} = x^3 + 9xy^2 - 3x$$

$$\Rightarrow \begin{cases} 3x^2 y + 3y^3 - 3y = 0 \\ x^3 + 9xy^2 - 3x = 0 \end{cases} \Leftrightarrow \begin{cases} x^2 y + y^3 - y = 0 \\ x^3 + 9xy^2 - 3x = 0 \end{cases}$$

$$\hookrightarrow x(x^2 + 9y^2 - 3) = 0$$

$$y=0, x=0 \text{ en } \begin{cases} x^2 + y^2 - 1 = 0 \\ x^2 + 9y^2 - 3 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x^2 + y^2 - 1 = 0 \\ 9y^2 - 3 = 0 \end{cases} \Leftrightarrow \begin{cases} x = \pm \sqrt{1 - y^2} \\ y = \pm \frac{\sqrt{3}}{3} \end{cases}$$

$$y=0 \rightarrow (x^2 + 0 - 3) = 0 \Leftrightarrow x = \pm \sqrt{3}$$

oef. 12 16 (a, b)

a) $f(x, y) = 2x^2 + 3y^2$; $(x + y = 1)$
 $\rightarrow$ mbw eliminatie

$\checkmark x + y = 1$

$\Leftrightarrow x = 1 - y$

$\checkmark f(1-y, y) = 2(1-y)^2 + 3y^2$
 $= 2(1 - 2y + y^2) + 3y^2$
 $= 2 - 4y + 2y^2 + 3y^2$
 $= 5y^2 - 4y + 2 = F(y)$

* extrema:

$F'(y) = 10y - 4 = 0$

$\Leftrightarrow y = \frac{4}{10} = \frac{2}{5}$

en $x = 1 - y = 1 - \frac{2}{5} = \frac{3}{5}$

b) $\rightarrow$ mbw. Lagrange techniek

$f(x, y) = 2x^2 + 3y^2$ en $g(x, y) = x + y = 1$

$\frac{\partial f}{\partial x} = 4x$; $\frac{\partial f}{\partial y} = 6y$; $\frac{\partial g}{\partial x} = 1$; $\frac{\partial g}{\partial y} = 1$

$\Rightarrow \begin{cases} 4x = \lambda \\ 6y = \lambda \\ x + y = 1 \end{cases} \Rightarrow \lambda = 4x = 6y$

$\Leftrightarrow 6y - 4x = 2(3y - 2x) = 0$

$\Leftrightarrow y = \frac{2x}{3}$ of $x = \frac{3y}{2}$

$$\Rightarrow y = \frac{2x}{3} \quad \& \quad x = \frac{3y}{2}$$

mit VW: $x + y = 1$

$$\hookrightarrow x + \frac{2x}{3} = 1 \quad \& \quad \frac{3y}{2} + y = 1$$

$$\Leftrightarrow \frac{5x}{3} = 1 \quad \& \quad \frac{5y}{2} = 1$$

$$\Leftrightarrow x = \frac{3}{5} \quad \& \quad y = \frac{2}{5}$$

$$\Rightarrow \text{stat. pkn: } \left(\frac{3}{5}, \frac{2}{5} \right)$$

oef 12 19 (a)

Tip: wat is de te minimaliseren functie?
 wat is de nevenvoorwaarde?

- a) $2x - y + 2z = 16 = g(x, y, z) \rightarrow$ nevenvw
 $\rightarrow$ ptn is deze opp die het dichtst bij oorspr. liggen

$\sqrt{x^2 + y^2 + z^2}$ is de te min functie
 $\hookrightarrow x^2 + y^2 + z^2 \Rightarrow$ minimaliseren $= f(x, y, z)$

(p. 311)

$\frac{\partial f}{\partial x} = 2x$	$\frac{\partial g}{\partial x} = 2$
$\frac{\partial f}{\partial y} = -2y$	$\frac{\partial g}{\partial y} = -1$
$\frac{\partial f}{\partial z} = 2z$	$\frac{\partial g}{\partial z} = 2$

$$\begin{cases} 2x = 2\lambda \\ 2y = -\lambda \\ 2z = 2\lambda \end{cases} \quad \Leftrightarrow \quad \lambda = \frac{2x}{2} = -2y = \frac{2z}{2}$$

$$\Leftrightarrow \lambda = x = -2y = z$$

$$\rightarrow \begin{cases} x + 2y = 0 \\ z + 2y = 0 \\ 2x - y + 2z = 16 \end{cases} \quad \Leftrightarrow \quad \begin{cases} y = -\frac{x}{2} \\ z + 2(-\frac{x}{2}) = 0 \\ 2x - (-\frac{x}{2}) + 2(-x) = 16 \end{cases} \quad \Leftrightarrow \quad \begin{cases} y = -\frac{x}{2} \\ z = x \\ -\frac{x}{2} - 2x = 16 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = -2y \\ z = -2y \\ y = 2x + 2z - 16 = 2(-2y) + 2(-2y) - 16 \end{cases} \quad \Leftrightarrow \quad \begin{cases} x = z \\ y = -4y - 16 \\ y = -8y - 16 \\ x = -2y \end{cases}$$

$$\Leftrightarrow \begin{cases} x = -2(-8) = 16 \\ y = -8 \\ z = 16 \end{cases} \quad \Leftrightarrow \quad \begin{cases} x = \frac{16}{5} \\ y = -\frac{16}{5} \\ z = \frac{16}{5} \end{cases}$$

$$\Leftrightarrow \begin{cases} x = \frac{16}{5} \\ y = -\frac{16}{5} \\ z = \frac{16}{5} \end{cases} \quad \Leftrightarrow \quad \begin{cases} x = \frac{16}{5} \\ y = -\frac{16}{5} \\ z = \frac{16}{5} \end{cases}$$

ex. 12.22 (a)

a) $f(x, y) = x^2 + y^2$
 $g(x, y) = x^4 + xy^4$
 $C = 1$

ander Nebenw $q(x, y) = C$

$$\frac{\partial f}{\partial x} = 2x$$

$$\frac{\partial Q}{\partial X} = 4X^3$$

$$\frac{21}{24} = \frac{7}{8}$$

$$\frac{\partial Q}{\partial y} = 4y^3$$

$$\begin{cases} 2x = 4\lambda x^3 \\ 2y = 4\lambda y^3 \\ x^4 + y^4 = 1 \end{cases}$$

$$\Rightarrow \lambda = \frac{\cancel{2} \times \cancel{2}}{\cancel{2} \times 32} = \frac{\cancel{2} \times \cancel{2}}{\cancel{2} \times 32}$$

$$= \frac{1}{2x^2} = \frac{1}{2y^2} \Leftrightarrow 2x^2 = 2y^2$$

$$\begin{cases} 2x^2 = 2y^2 \\ x^4 + y^4 = 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} x^2 = 1 \\ x^4 + (x^2)(x^2) = 1 \end{cases}$$

$$\Leftrightarrow 2x^4 = 1 \quad \Leftrightarrow x^4 = \frac{1}{2} \quad \Leftrightarrow x = \frac{+1}{\sqrt[4]{2}}$$

$$\Rightarrow y = \pm \frac{1}{\sqrt{2}}$$

$$\ln x = 0$$

(met vw:)

en $y = 0$ (je $\mathbb{K}$ ml delen door 0)

$$\rightarrow M = \pm 1 \quad \rightarrow X = \pm 1$$

8 stat. ptn.

8 stat. ptn: $(0, 1); (0, -1); (1, 0); (-1, 0);$
 $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}); (\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}); (\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}); (\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

oef 12.28 (b)

b) $V = I(R+x) \xrightarrow{IR+IX}$ nevenw. Ven $R = \text{cte}$
 $x=R$
 $y = I^2 x = f(x, I)$
Gevr: voor welke $x \geq 0$ is $y = \dots$ bij variabele $R = \max$

$$\frac{\partial f}{\partial x} = I^2$$

$$\frac{\partial g}{\partial x} = I$$

$$\frac{\partial f}{\partial I} = 2Ix$$

$$\frac{\partial g}{\partial I} = R+x$$

$$\Rightarrow \begin{cases} I^2 = \lambda I \\ 2Ix = \lambda(R+x) \\ V = IR + Ix \end{cases}$$

$$\Rightarrow \lambda = I = \frac{2Ix}{R+x}$$

$$\Rightarrow \begin{cases} I(R+x) = 2Ix \\ V = IR + Ix = I(R+x) \end{cases}$$

$$\Leftrightarrow R+x = 2x \Rightarrow \boxed{R=x}$$

oef 12.29

Tip: e^{2xy} maximaal $\Leftrightarrow 2xy$ maximaal
(want e^t is stijgend)
 $\Rightarrow$ maximaliseren $2xy$ (eenvoudiger)

$f(x, y) = e^{2xy} \Rightarrow 2xy$
nevenw: $x^3 + 8y^3 = 16$ met $x \geq 0$ en $y \geq 0$

$$\frac{\partial f}{\partial x} = 2y$$

$$\frac{\partial g}{\partial x} = 3x^2$$

$$\frac{\partial f}{\partial y} = 2x$$

$$\frac{\partial g}{\partial y} = 24y^2$$

$$\begin{cases} 2y = 3\lambda x^2 \\ 2x = 24\lambda y^2 \\ x^3 + 8y^3 = 16 \end{cases}$$

oef. 13.3 (a, b)

a) $P(1, -2, 0)$ en $Q(4, 2, 0)$

$\rightarrow \vec{a} = \vec{PQ}$

$\|\vec{a}\| = \|\vec{PQ}\| = \sqrt{(1-4)^2 + (-2-2)^2 + (0-0)^2} = 5$

$\rightarrow$ eenheidsvector $\vec{a} = (3, 4, 0)$

"lengte" = 1

$\Rightarrow (\frac{3}{5}, \frac{4}{5}, 0)$

b) $P(-3, 2, 1)$ en $Q(-1, -3, 2)$

$\rightarrow \vec{a} = (2, -5, 1)$

$\|\vec{a}\| = \sqrt{4 + 25 + 1} = \sqrt{30}$

$\rightarrow$ eenheidsvector

$\hookrightarrow (\frac{2}{\sqrt{30}}, \frac{-5}{\sqrt{30}}, \frac{1}{\sqrt{30}})$

oef. 13.4

$\vec{a} = (1, 2, 3)$

$\vec{b} = (-2, 3, -4)$

$\vec{c} = (0, 4, -1)$

c) $3\vec{a} + 2\vec{b} - \vec{c} = (-1, 8, 2)$

$\rightarrow 3\vec{a} = (3, 6, 9)$

$\rightarrow 2\vec{b} = (-4, 6, -8)$

a) $\vec{a} + \vec{b} = (-1, 5, -1)$

$\vec{a} - \vec{b} = (3, -1, 7)$

d) $3(\vec{a} - \vec{c}) = 3(1, -2, 4) = (3, -6, 12)$

$3\vec{a} - 3\vec{c} = (3, -6, 12)$

$\rightarrow 3\vec{c} = (0, 12, -3)$

b) $3\vec{a} = (3, 6, 9)$

$-\vec{a} = (-1, -2, -3)$

$\vec{a}/3 = (\frac{1}{3}, \frac{2}{3}, 1)$

e) $|\vec{a} + \vec{b}| = 3\sqrt{3} = \sqrt{27}$

f) $|\vec{a}| + |\vec{b}| = \sqrt{14} + \sqrt{29}$

oef. 13.9 (a)

$$a) \vec{a} \cdot \vec{b} = \\ 0 + 9 + (-2) = 7$$

$$\vec{b} \cdot \vec{a} = 7$$

$$\begin{cases} \vec{a} = (1, 3, -2) \\ \vec{b} = (0, 3, 1) \\ \vec{c} = (1, -1, -3) \end{cases}$$

oef. 13.11

$$\begin{cases} \vec{a} = (1, 3, 1) \\ \vec{b} = (2, 1, -1) \end{cases} \quad \vec{a} \cdot \vec{b} = 0$$

$$\Leftrightarrow 2\lambda + 3 - 1 = 0 \quad \Leftrightarrow \lambda = -1$$

oef. 13.19 (a, b)

$$K: x^3 - 3x^2 + 4y^2 = 0$$

a) raaklijn aan K in punt $(-1, 1)$

$$\frac{\partial K}{\partial x} = 3x^2 - 6x \quad \leadsto \text{in } x = -1 \rightarrow 3(-1)^2 - 6(-1) = 9$$

$$\frac{\partial K}{\partial y} = 8y \quad \leadsto \text{in } y = 1 \rightarrow 8(1) = 8$$

$$\Rightarrow 9(x+1) + 8(y-1) = 0$$

$$\Leftrightarrow 9x + 9 + 8y - 8 = 0$$

$$\Leftrightarrow 8y = -9x - 1$$

$$\Leftrightarrow y = -\frac{9}{8}x - \frac{1}{8}$$

$$\left[\frac{\partial f}{\partial x} (x - x_0) + \frac{\partial f}{\partial y} (y - y_0) \right]$$

b) ngl. rechte door $(-1, 1) \perp$ op K

$$\frac{\partial K}{\partial x} = 3x^2 - 6x \leadsto \text{in } x = -1 \Rightarrow 9$$

$$\frac{\partial K}{\partial y} = 8y \leadsto \text{in } y = 1 \Rightarrow 8$$

$$\left. \begin{array}{l} \frac{\partial K}{\partial x} = 9 \\ \frac{\partial K}{\partial y} = 8 \end{array} \right\} \frac{dy}{dx} = \frac{8}{9}$$

$$\Leftrightarrow y - 1 = \frac{8}{9}(x + 1)$$

$$\Leftrightarrow y = \frac{8}{9}x + \frac{17}{9}$$

oef. 13.20 (a, b)

$$f(x, y) = \sin x - \cos y$$

$$\vec{x} = \frac{1}{5}(4, 3)$$

a) richtinggr. v. f in $P(\frac{\pi}{4}, \frac{\pi}{4})$ id. richting v. e

$$\star \text{grad } f\left(\frac{\pi}{4}, \frac{\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$$\frac{\partial f}{\partial x} = \cos x \quad ; \quad \frac{\partial f}{\partial y} = \sin y$$

$$\text{grad } f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$$

$$\star D_{\vec{e}} f = \text{grad } f \cdot \vec{e}$$

$$= \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \cdot \left(\frac{4}{5}, \frac{3}{5}\right)$$

$$= \frac{2\sqrt{2}}{5} + \frac{3\sqrt{2}}{10} = \frac{7\sqrt{2}}{10}$$

b) grootste id. richting v. d. grad. = 1

oef 13.7

$$\vec{f}_1 + \vec{f}_2 + \vec{f}_3 = \vec{0}$$

$$\vec{f}_1 = (2, -3, 1)$$

$$\vec{f}_3 = (0, 2, -1)$$

$$\Rightarrow \vec{f}_2 = (-2, 1, 0)$$

oef 13.9 (b, c, d)bgecos($\frac{7}{\sqrt{10}}$)

$$\vec{a} = (1, 3, -2)$$

$$\vec{b} = (0, 3, 1)$$

$$\vec{c} = (1, -1, -3)$$

$$b) (\vec{a} - \vec{b}) \cdot \vec{c}$$

$$\Rightarrow (\vec{a} - \vec{b}) = (1, 0, -3)$$

$$\Leftrightarrow (\vec{a} - \vec{b}) \cdot \vec{c} = (1 + 0 + 9) = 10 = (\vec{a} \cdot \vec{c}) - (\vec{b} \cdot \vec{c})$$

$$c) (\vec{a} \cdot \vec{c}) \cdot \vec{b}$$

$$\Rightarrow \vec{a} \cdot \vec{c} = 1 - 3 + 6 = 4$$

$$\Rightarrow (\vec{a} \cdot \vec{c}) \vec{b} = (0, 12, 4)$$

d) hoek tm $\vec{a}$ en $\vec{b}$

$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \cdot \|\vec{b}\| \cdot \cos \theta$$

$$\Leftrightarrow \theta = \arccos \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|}$$

$$\rightarrow \vec{a} \cdot \vec{b} = 0 + 9 - 2 = 7$$

$$\|\vec{a}\| = \sqrt{1+9+4} = \sqrt{14}$$

$$\|\vec{b}\| = \sqrt{0+9+1} = \sqrt{10}$$

$$\theta = \arccos \frac{7}{\sqrt{140}}$$

oef. 13.15(a,b)

$$\vec{r}(t) = (at, at - \frac{1}{2}gt^2)$$

a) $\vec{v}(t) = (a, a - gt)$
 $\vec{a}(t) = (0, -g)$

b) $(0, -mg)$

oef. 13.19(c)

Tip: zie paragraaf 13.5.2 (p. 335)

c) raaklijn // x-as of y-as
→ bepaal ptn K

$$K: x^3 - 3x^2 + 4y^2 = 0$$

$$[\text{grad } f(K) \cdot (\vec{x} - \vec{x}_0) = 0]$$

$$\text{grad } f = (3x^2 - 6x, 8y)$$

✓ // x-as → $\frac{\partial f}{\partial x} = 0$

$$\Rightarrow 3x^2 - 6x = 0$$

$$\Leftrightarrow x(3x - 6) = 0$$

$$x = 0$$

~~$\Rightarrow (0, 2y) \text{ met } \vec{x} - \vec{x}_0 = 2y$~~

$$\text{en } x = 2$$

↪ enkel op houden
waarvoor $\frac{\partial f}{\partial y} \neq 0$

$x = 0$ en $x = 2$ invullen in K:

$$\underline{x=0}: 0^3 - 3(0)^2 + 4y^2 = 0$$

$$\Leftrightarrow 4y^2 = 0 \Leftrightarrow y^2 = 0 \Leftrightarrow y = 0$$

$$\underline{x=2}: (2)^3 - 3(2)^2 + 4y^2 = 0$$

$$\Leftrightarrow 8 - 12 + 4y^2 = 0$$

$$\Leftrightarrow 2 - 3 + y^2 = 0$$

$$\Leftrightarrow y^2 = 1 \Leftrightarrow y = \pm 1$$

✓ // y-as → $\frac{\partial f}{\partial y} = 0$

$$\Rightarrow 8y = 0$$

$$\Leftrightarrow y = 0$$

$$\underline{y=0}: x^3 - 3x^2 + 0 = 0$$

$$\Leftrightarrow x^2(x - 3) = 0 \Leftrightarrow x = 0 \text{ en } x = 3$$

$$\Rightarrow // \text{x-as}$$

$$\hookrightarrow (0, 0);$$

$$(2, 1); (2, -1)$$

$$// \text{y-as}$$

$$\hookrightarrow (0, 0); (3, 0)$$

oef. 13.20 (c)

$$f(x, y) = \sin x - \cos y \text{ en } \vec{r} = \frac{1}{5} (4, 3)$$

$$c) \frac{\partial f}{\partial x} = \cos x \rightarrow \text{in } a: \cos a$$

$$\frac{\partial f}{\partial y} = \sin y \rightarrow \text{in } b = \sin b$$

$$\star D_{\vec{r}} f = (\cos a, \sin b) \left(\frac{4}{5}, \frac{3}{5} \right)$$

$$= \frac{4}{5} \cos a + \frac{3}{5} \sin b = \frac{1}{5} (4 \cos a + 3 \sin b)$$

$$\star \text{ wanneer } \cos a = 1 \left. \begin{array}{l} \sin b = 1 \end{array} \right\} \text{ max.}$$

$$\left. \begin{array}{l} a = 2\pi k \\ b = \frac{\pi}{2} + 2\ell\pi \end{array} \right\} (2\pi k, \frac{\pi}{2} + 2\ell\pi) \text{ met } k, \ell \in \mathbb{Z}$$

oef. 13.17 (c, d, e)

$$\vec{r}(t) = (at - b \sin t, a - b \cos t)$$

$$\hookrightarrow 1 + \cos 2\theta = 2 \cos^2 \theta$$

$$1 - \cos 2\theta = 2 \sin^2 \theta$$

$$c) \|\vec{r}'(t)\| = \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}$$

$$\star \vec{r}'(t) = (a - b \cos t, b \sin t)$$

$$\star \|\vec{r}'(t)\| = \sqrt{(a - b \cos t)^2 + (b \sin t)^2}$$

$$= \sqrt{a^2 - 2ab \cos t + b^2 \cos^2 t + b^2 \sin^2 t} \quad (= \cos^2 t + \sin^2 t = 1)$$

$$= \sqrt{a^2 - 2ab \cos t + b^2}$$

d) lengte omwenteling bij apt. b

volledige $\rightarrow 2\pi$ (180° omwentelen)

$$\Rightarrow \int_0^{2\pi} \sqrt{a^2 - 2ab \cos t + b^2} dt$$

$$e) \int_0^{2\pi} \sqrt{a^2 - 2a^2 \cos t + a^2} dt$$

$$\Leftrightarrow \frac{1 - \cos 2\theta = 2\sin^2 \theta}{\sqrt{2}} = 2\sin \theta$$

$$= \int_0^{2\pi} \sqrt{2a^2 - 2a^2 \cos t} dt = \int_0^{2\pi} \sqrt{2a^2 (1 - \cos t)} dt$$

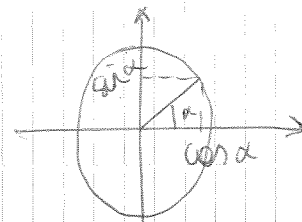
$$= a\sqrt{2} \int_0^{2\pi} \sqrt{(1 - \cos t)} dt = a\sqrt{2} \int_0^{2\pi} \left(\sqrt{2} \sin\left(\frac{t}{2}\right) \right) dt$$

$$= 2a \int_0^{2\pi} \sin\left(\frac{t}{2}\right) dt = 2a \left[-2\cos\left(\frac{t}{2}\right) \right]_0^{2\pi}$$

$$= 2a [+2 - (-2)] = 8a$$

$$\frac{t}{2} = u$$

$$\frac{1}{2} dt = du$$

① Def.

$$\checkmark \sin 0 = 0$$

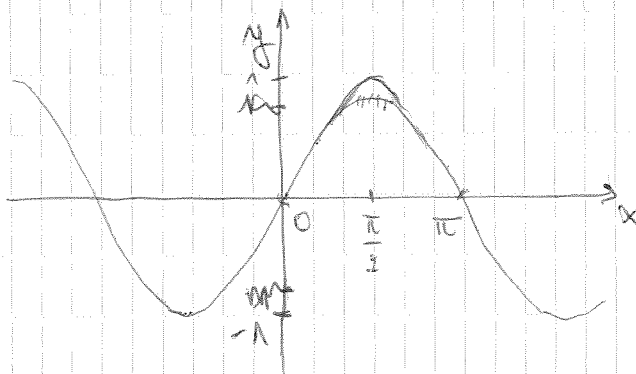
$$\checkmark \sin \frac{3\pi}{2} = -1$$

$$\checkmark f(x) = \sin x$$

$$\checkmark \sin \frac{\pi}{2} = 1$$

$$\checkmark \sin 2\pi = 0$$

$$\checkmark \sin \pi = 0$$



$$\checkmark \sin x = 0 \Leftrightarrow x = (\pi + 1)\pi k \quad k \in \mathbb{Z} \quad [x \in \{k \cdot \pi \mid k \in \mathbb{Z}\}]$$

$$\checkmark \sin x = 1 \Leftrightarrow x = \frac{\pi}{2} + 2\pi k \quad k \in \mathbb{Z} \quad [x \in \{\frac{\pi}{2} + k\pi \mid k \in \mathbb{Z}\}]$$

$$② \quad 1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha}$$

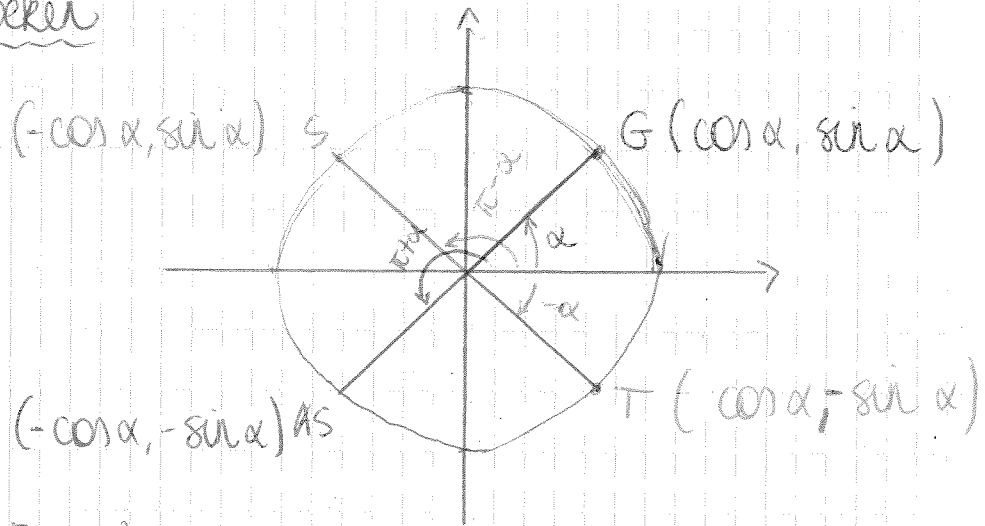
$$\sin^2 \alpha + \cos^2 \alpha = 1 \quad (\text{HF})$$

↪ alles $\cdot \frac{1}{\cos^2 \alpha}$

③

α	0	30° " $\pi/6$ "	45° " $\pi/4$ "	60° " $\pi/3$ "	$\pi/2 = 90^\circ$
$\sin \alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos \alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan \alpha$	0	$\frac{\sqrt{3}}{3}$ " $\frac{1}{\sqrt{3}}$ "	1	$\sqrt{3}$	/

④ verwante hoeken



$$\rightarrow \text{CH} \quad \sin\left(\frac{\pi}{2} - \alpha\right) = \cos \alpha$$

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin \alpha$$

⑤ oef

$$\checkmark \sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2} ; \cos \frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\checkmark \sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} ; \cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$X \checkmark \sin\left(-\frac{28\pi}{3}\right) = \sin -\frac{\pi}{3} = -\frac{\sqrt{3}}{2} ; \cos -\frac{\pi}{3} = \frac{1}{2}$$

$$\checkmark \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} ; \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\checkmark \sin \frac{5\pi}{6} = \frac{1}{2} ; \cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$\frac{4\pi}{3} = \frac{\pi}{3} + \pi$$

$$\left(-\frac{2\pi}{3}\right) + 2\pi = \frac{4\pi}{3}$$

~~scribbles~~

~~scribbles~~

~~scribbles~~

$$\binom{n}{k}$$

$$= \frac{n!}{k!(n-k)!}$$