Relativity exam

25 January 2022

Theoretical question

(10 points)

Note: This is a question that would suite an oral with written preparation part of the exam. As this is a completely written exam try not to be too elaborate going into long derivations. Your answers should be contained to 2-3 pages.

Why are Killing vectors important in General Relativity? Discuss one (or more) applications that we saw during the lectures. What is a Komar integral? What are they used for?

Problem 1

(3 points)

Given the de Sitter metric. Give explicitly the Killing vectors.

$$ds^2 = -d\tau^2 + cosh^2(\tau)d\xi^2$$

(Hint: De Sitter is a maximally symmetric space-time)

Problem 2

(7 points)

We consider wormholes. More specifically the Ellis wormhole. The metric that we are here considering is given.

$$ds^2 = -dt^2 + d\rho^2 + (\rho^2 + n^2)(d\theta^2 + sin^2\theta d\phi^2)$$

Here n is a real constant and the coordinates on the space time are $x^\mu = (t, \rho, \theta, \phi)$.

1. Consider $n = 0$. What is the resulting space-time? What is the range of ρ? Why? What is the locus at $\rho = 0$ and time $t = const.$ constant. How much dimensions does this locus have? Why?

2. Consider $n \neq 0$. What is now the locus at $\rho = 0$ and time $t = const.$ constant? What are the dimensions of this locus and why? Describe what happens for $\rho > 0$ large and why. Same thing for $\rho < 0$ large and negative. Based on your answer in 1. is $\rho < 0$ even allowed? Why, Why not?

3. Draw a cartoon for what happens at $\rho < 0$, $\rho = 0$ and $\rho > 0$ and how these asymptotically regions are connected. You have (hopefully) just drawn a wormhole.

4. Given the Einstein equation (also on formula sheet) with the energy momentum tensor for at this point yet to be determined fields, we do not know what the right hand side will look like. Consider the left hand side, the Einstein tensor. Compute it's tt component, thus $G_{tt} =?$. What does this value mean for T_{tt}, the tt component of the energy momentum tensor? Is this a physical result?