Final Examination Star and Planet Formation
17 Aug 2021, 12:00-15:00

Important: This examination takes three hours. Divide your time in such a way that you have sufficient time for each question. In answering, use complete and grammatically correct sentences. The use of a pocket calculator is allowed. State on at least one sheet your student number and state your name on *each* sheet.

Points: Q1 or Question 1: 4.0/20; Q2: 4.0/20; Q3: 4.0/20; Q4: 4.0/20; Q5: 3.0/20; Q6: 2.0/20

1. Discuss and/or explain the following concepts:

a) Star-formation history of the universe
b) Ambipolar diffusion
c) Hayashi tracks
d) Gravitational focussing
e) Isolation mass

2. Describe the four distinct phases that can be identified in the formation of solar type stars. Point out which phases are isothermal and which are not, and in which phase the star accretes most of its mass. Explain which physical process causes the star formation process to 'switch' to the next phase, and discuss the associated typical time-scale of the first and last phase.

3. We consider a protostar of radius $R = 2\,\mathrm{R}_\odot$ and mass $M = 1\,\mathrm{M}_\odot$ that through radial infall accretes material at a rate of $\dot{M}_{\rm acc} = 10^{-5} M_\odot \mathrm{yr}^{-1}$. Recall that $1 M_\odot \mathrm{yr}^{-1} = 6.303 \times 10^{25}\,\mathrm{gr\,s}^{-1}$. The luminosity that it generates consists of two components. The accretion of material in a shock at the protostellar surface generates an accretion luminosity

$$L_{\rm acc} = \frac{G \dot{M}_{\rm acc} M}{R} \quad \mathrm{erg\,s}^{-1} \qquad (1)$$

The protostar also generates luminosity through deuterium burning in its core. We assume that this luminosity is given by its steady-state value

$$L_{\rm D} = 12 \left(\frac{\dot{M}_{\rm acc}}{10^{-5}\, M_\odot \mathrm{yr}^{-1}} \right) \quad \mathrm{L}_\odot. \qquad (2)$$

a) Explain the concept of steady-state deuterium burning in a protostar.

One cannot directly observe the protostar as it is surrounded by an in-falling envelope (from which it accretes its material) that sufficiently far from the protostar contains solid state (i.e. dust) particles. This dust envelope is optically thick. We aim to calculate the radius and temperature of the outside of this dust envelope, at $R_{\rm phot}$, which we will call the dust photosphere. We adopt a mean dust extinction of $\kappa_\circ = 2.8\,{\rm cm^2\,gr^{-1}}$. An increment in optical depth is given by $d\tau = \kappa_\circ\,\rho\,dr$, where ρ is the total density of the material and dr an increment in the radial distance to the center of the protostar. At the location of the dust envelope the gas is in free fall, which results in a density profile

$$\rho(r) = \frac{\dot{M}_{\rm acc}}{4\pi\sqrt{2GM}}\frac{1}{r^{3/2}}, \tag{3}$$

where G is the gravitational constant.

b) Assume the dust photosphere to be at the layer where $\tau = 2/3$, where the optical depth is measured from the outside in. Derive an expression for the radius of the dust photosphere in terms of $\kappa_\circ$, $\dot{M}_{\rm acc}$, and M. Compute the radius of the dust photosphere and express your answer in AU.

c) Compute the temperature $T_{\rm phot}$ of the dust photosphere in Kelvin. In which part of the electromagnetic spectrum will this source emit its observable light?

d) We assume that the protostar is very near the end of the main accretion phase and that the mass accretion rate has dropped to $10^{-6} M_\odot {\rm yr}^{-1}$ throughout the entire envelope (so, from infinity to R). The values of M and R have not changed compared to the prior high mass accretion phase, and the dust envelope is still optically thick. Will $T_{\rm phot}$ have gone up or down? Explain your answer.

4. We consider the outcomes of the collision of objects of a range in sizes and masses. The impactor (the projectile) has mass $m_{\rm p}$. The main body (the target) has mass $M_{\rm t}$ and radius $R_{\rm t}$. The outcome of a collision may be described quantitatively using the specific energy of the impact

$$Q \equiv \frac{m_{\rm p} v^2}{2M_{\rm t}}, \tag{4}$$

where v is the impact velocity. For Q larger than the disruption threshold $Q^*_{\rm D}$ the impact is catastrophic and fragments the target into many pieces that do not remain gravitationally bound. For a projectile smashing into a target the disruption threshold of the target body can be described as

$$Q^*_{\rm D} = Q_0 \left(\frac{R_{\rm t}}{1\,{\rm cm}}\right)^a + B\,\rho \left(\frac{R_{\rm t}}{1\,{\rm cm}}\right)^b, \tag{5}$$

where $R_{\rm t}$ is the radius of the target body. $Q_0 = 3.5 \times 10^7\ {\rm erg\,gr^{-1}}$ and $B = 0.3\ {\rm erg\,cm^3\,gr^{-2}}$. The power-law indice $a = -0.38$ and $b = 1.36$. ρ is the mean density of the target body. We adopt $\rho = 5.5\,{\rm gr\,cm^{-3}}$, typical for iron-rich basaltic material.

a) This functional form shows that $Q^*_{\rm D}$ is described by two distinct physical regimes, represented by the two terms. Give the terms describing these physical regimes and explain the behavior $Q^*_{\rm D}(R_{\rm t})$ in each of these regimes.

b) Derive an expression that establishes the radius $R_{\rm weak}$ of the most vulnerable target bodies and compute $R_{\rm weak}$ for the iron-rich basaltic material discussed here.

Probably the moon formed from debris ejected into an Earth-orbiting disk by the collision of a large planet with the early Earth. Canup (2012) presents smooth-particle-hydrodynamics simulations of this event, finding that a projectile of mass 0.416 $M_\oplus$ smashing into an early Earth of mass 0.624 $M_\oplus$ yields an event that can explain essential similarities and differences in the composition of the current moon and Earth.

c) Compute an upper limit to the impact velocity of this event, i.e. the maximum velocity for the event not to lead to catastrophic disruption.

d) Whether or not an impact leads to catastrophic disruption is not solely determined by $m_{\rm p}/M_{\rm t}$, v, and compositional properties. Give three other properties that may control the outcome. (Feel free to add a drawing to explain things better).

5. In the theory of planet formation through the growth of grains, four distinct stages can be identified. Describe the dust growth mechanism that is operating in each of these phases, and provide typical timescales for the first two phases. Which phase is at the present time theoretically most problematic and why?

6. True or false?

a) Star formation may occur in any of the thermal phases of the interstellar medium.

b) Reflection nebulae are clouds in which the dust particles reflect the light of a nearby star or stars.

c) In the proto-stellar phase the proto-stellar interior cannot thermally adjust to the accretion and the luminosity will be generated by accretion shocks on the surface.

d) It is often assumed that weak-lined T Tauri stars are the younger counterparts of the classical T Tauri stars.

e) The majority of massive stars (defined as having an initial mass of at least $10\,{\rm M}_\odot$) have likely been formed through the coalescence of low- and intermediate-mass stars.

f) In a non-viscous proto-planetary disk it would be impossible to transport angular momentum.

g) A razor thin, optically thick proto-planetary disk that extends from the surface to infinity intercepts a negligibly small fraction of the stellar light.

h) The final stage in terrestrial planet formation, that of late impacts during final assembly, is the longest (in terms of time) of all phases.

i) The Hill sphere around a planet will be larger if it is in an eccentric orbit (compared to it being in a circular orbit).

j) The total mass of planetesimals in the feeding zone around a planetary embryo fixes the maximum attainable mass that the embryo can reach, called the Hill mass.

Formulae

$$\tau_{\rm ff} = \left(\frac{3\pi}{32}\frac{1}{G\rho}\right)^{1/2} = 3.4\times10^5\left(\frac{10^4\,{\rm cm^{-3}}}{n}\right)^{1/2}\quad{\rm yr},$$

= free-fall time, assuming a mean molecular weight $\mu = 2.3$ in the second equality

$$\tau_{\rm KH} = \frac{E_{\rm pot}}{L} \equiv \frac{G M^2}{R L} = 3\times10^7\left(\frac{M}{1\,{\rm M_\odot}}\right)^2\left(\frac{R}{1\,{\rm R_\odot}}\right)^{-1}\left(\frac{L}{1\,{\rm L_\odot}}\right)^{-1}\quad{\rm yr},$$

= Kelvin-Helmholtz contraction time

$$2E_{\rm kin} + 2E_{\rm th} + E_{\rm mag} + E_{\rm grav} = 0$$

= virial theorem, expressing a global dynamical equilibrium of kinetic, thermal, magnetic, and gravitational potential energy.

$$E_{\rm grav} = -\frac{3}{5}\frac{G M^2}{R},$$

= gravitational potential of a homogeneous sphere

$$L_{\rm acc} = \frac{GM\dot{M}}{R} = 62.8\ L_\odot\left(\frac{\dot{M}}{10^{-5}\ {\rm M_\odot\ yr^{-1}}}\right)\left(\frac{M}{1\,{\rm M_\odot}}\right)\left(\frac{R}{5\,{\rm R_\odot}}\right)^{-1}$$

= accretion luminosity assuming all of the kinetic (infall) energy is converted into radiation

$$L_{\rm phot} = 4\pi R_{\rm phot}^2\sigma T_{\rm phot}^4,$$

= photospheric luminosity of spherical object

$$M > M_{\rm J} = \left(\frac{5kT}{G\,\mu\, m_{\rm H}}\right)^{3/2}\left(\frac{3}{4\pi\,\rho}\right)^{1/2} = 7.41\left(\frac{T}{10\,{\rm K}}\right)^{3/2}\left(\frac{2}{\mu}\right)^2\left(\frac{10^4}{n}\right)^{1/2}\ {\rm M_\odot}$$

= Jeans instability criterion

$$\dot{M} = 0.975\,\frac{a^3}{G} = 1.6\times10^{-6}\left(\frac{T}{10\,{\rm K}}\right)^{3/2} M_\odot{\rm yr^{-1}},$$

= mass accretion rate in Frank Shu's inside-out collapse solution

$$\Omega = \frac{v_{\rm rot}}{r} = \sqrt{\frac{GM}{r^3}}\qquad\text{angular velocity}$$

$$\nu = \alpha\, a\, h$$

= turbulent viscosity, where h is the scale height and a is the sound speed given by

$$a = \sqrt{\frac{kT}{\mu\, m_{\rm H}}}$$

$$\frac{dm}{dt} = \pi s^2\, v\,\rho$$

= mass accretion rate of a particle of size s that is moving at speed v through a medium of density ρ

Physical Constants

Name	Symbol	CGS Value	CGS units
Speed of light in a vacuum	c	2.998×10^{10}	$\mathrm{cm\,s^{-1}}$
Planck constant	h	6.626×10^{-27}	$\mathrm{erg\,s}$
	$\hbar$	1.055×10^{-27}	$\mathrm{erg\,s}$
Gravitational constant	G	6.673×10^{-8}	$\mathrm{cm^3\,g^{-1}\,s^{-2}}$
Atomic mass unit	$\mathrm{m_{amu}}$	1.661×10^{-24}	g
Mass of hydrogen	$\mathrm{m_H}$	1.673×10^{-24}	g
Mass of proton	$\mathrm{m_p}$	1.673×10^{-24}	g
Mass of neutron	$\mathrm{m_n}$	1.675×10^{-24}	g
Mass of electron	$\mathrm{m_e}$	9.109×10^{-28}	g
Avagadro's number	N_A	6.022×10^{23}	
Boltzmann constant	k	1.381×10^{-16}	$\mathrm{erg\,K^{-1}}$
Electron volt	eV	1.602×10^{-12}	erg
Radiation density constant	a	7.565×10^{-15}	$\mathrm{erg\,cm^{-3}\,K^{-4}}$
Stefan-Boltzmann constant	σ	5.671×10^{-5}	$\mathrm{erg\,cm^{-2}\,K^{-4}}$
Gas constant	$\mathcal{R}$	8.314×10^{7}	$\mathrm{erg\,K^{-1}\,mol^{-1}}$

Astronomical Constants

Name	Symbol	CGS Value	CGS units
Astronomical unit	AU	1.496×10^{13}	cm
Parsec	pc	3.086×10^{18}	cm
Light year	ly	9.463×10^{17}	cm
Earth radius	$R_\oplus$	6.371×10^{8}	cm
Earth mass	$M_\oplus$	5.972×10^{27}	g
Solar mass	$\mathrm{M_\odot}$	1.989×10^{33}	g
Solar luminosity	$\mathrm{L_\odot}$	3.828×10^{33}	$\mathrm{erg\,s^{-1}}$
Solar radius	$\mathrm{R_\odot}$	6.957×10^{10}	cm
Solar effective temperature	$T_{\mathrm{eff},\odot}$	5 772	K
Thomson scattering coefficient	σ_T	6.652×10^{-25}	$\mathrm{cm^2}$

Conversion Factors

Name	Symbol	Value	CGS units
Year	yr	3.156×10^{7}	s
Arcsec	"	4.848×10^{-6}	radians
Solar mass per year	$\mathrm{M_\odot\,yr^{-1}}$	6.303×10^{25}	$\mathrm{g\,s^{-1}}$