DIFFUSION FROM TWO SOURCES

$$c(x,t) = \frac{1}{\sqrt{4\pi D t}} \left\{ \frac{3N}{4} e^{-\frac{(x+a)^2}{4Dt}} + \frac{N}{4} e^{-\frac{\left(x-\frac{a}{2}\right)^2}{4Dt}} \right\}$$

CURRENT : $j(x,t) = -D \frac{\partial c}{\partial x} = \frac{-D}{\sqrt{4\pi D t}} \frac{N}{4} \left\{ -3 \frac{2(x+a)}{4Dt} e^{-\frac{(x+a)^2}{4Dt}} \right.$

$$\left. - \frac{2\left(x-\frac{a}{2}\right)}{4Dt} e^{-\frac{(x-a/2)^2}{4Dt}} \right\}$$

$$j(0,t) = \underset{\substack{\uparrow \\ \text{PREFACTORS}}}{\cdots\cdots} \left\{ 6\, e^{-\frac{a^2}{4Dt}} - e^{-\frac{a^2}{16Dt}} \right\}$$

DEFINE $X \equiv e^{-\frac{a^2}{16Dt}}$ $\qquad 6X^4 - X = 0$

$X = 0 \qquad \boxed{\tau \to \infty}$

$X = \left(\frac{1}{6}\right)^{1/3} \qquad -\frac{a^2}{16 D \tau} = -\frac{1}{3} \log 6 \qquad \boxed{\tau = \frac{3a^2}{16 D \log 6}}$

GAS IN A GRAVITATIONAL FIELD

$$\rho(\vec{r}) = N \langle \delta(\vec{r} - \vec{q}_1) \rangle = \frac{N \int d\vec{q}_1\, \delta(\vec{r} - \vec{q}_1)\, e^{-\beta m g z_1}}{\int d\vec{q}_1\, e^{-\beta m g z_1}} =$$

$$= \frac{N e^{-\beta m g z}}{\pi R^2 \int_0^L e^{-\beta m g z_1} d z_1} = \frac{N \beta m g}{\pi R^2} \frac{e^{-\beta m g z}}{1 - e^{-\beta m g L}}$$

THE CORRECT ANSWER IS c). VELOCITIES AND POSITIONS ARE INDEPENDENT VARIABLES

KINETIC ENERGY OF CLASSICAL PARTICLE

SPEED DISTRIBUTION $\tilde{p}(v) = \cdots v^2 e^{-\beta \frac{m v^2}{2}}$

NORMALIZATION

$$E_k = \frac{m v^2}{2} \qquad p(E_k) = \tilde{p}(v)\,\frac{dv}{dE_k}$$

$$v = \sqrt{\frac{2E_k}{m}} \qquad \frac{dv}{dE_k} = \cdots \frac{1}{\sqrt{E_k}} \implies p(E_k) = \cdots \frac{E_k\, e^{-\beta E_k}}{\sqrt{E_k}}$$

$$\langle E_k \rangle = \frac{\int dE_k\, E_k^{3/2}\, e^{-\beta E_k}}{\int dE_k\, E_k^{1/2}\, e^{-\beta E_k}} = -\frac{\partial}{\partial \beta} \log\left(\int_0^{+\infty} dE_k\, E_k^{1/2}\, e^{-\beta E_k}\right)$$

CHANGE VAR. $x \equiv \beta E_k$

$$= -\frac{\partial}{\partial \beta} \log\left[\beta^{-3/2} \int_0^{+\infty} dx\, x^{1/2}\, e^{-x}\right] = \frac{3}{2\beta} = \frac{3 k_B T}{2}$$

AS EXPECTED FROM EQUIPART.

MOST PROBABLE KINETIC ENERGY IS THE MAX OF $p(E_k)$

$$\frac{dp(E_k)}{dE_k} = \cdots \left(\frac{1}{2\sqrt{E_k}} - \beta\sqrt{E_k}\right) e^{-\beta E_k} = 0 \implies \boxed{E_k^* = \frac{k_B T}{2}}$$

ANHARMONIC OSCILLATOR

$$Z = \int dx\, e^{-\beta\left(\frac{kx^2}{2} + \frac{\Gamma x^4}{4}\right)} \approx \int dx\, e^{-\beta\frac{kx^2}{2}} \left(1 - \beta\frac{\Gamma x^4}{4}\right) =$$

$$= \left(1 - \frac{\beta\Gamma}{\cancel{4}}\frac{\cancel{4}}{\beta^2}\frac{\partial^2}{\partial k^2}\right) \int dx\, e^{-\beta\frac{kx^2}{2}} = \left(1 - \frac{\Gamma}{\beta}\frac{\partial^2}{\partial k^2}\right)\sqrt{\frac{2\pi}{\beta k}} =$$

$$= \left(1 - \frac{\Gamma}{\beta}\,\frac{1}{2}\cdot\frac{3}{2}\,\frac{1}{k^2}\right)\sqrt{\frac{2\pi}{\beta k}} = \sqrt{\frac{2\pi}{\beta k}}\left(1 - \frac{3\Gamma}{4\beta k^2}\right)$$

$$\log Z = \frac{1}{2}\log\left(\frac{2\pi}{\beta k}\right) + \log\left(1 - \frac{3\Gamma}{4\beta k^2}\right) \approx \frac{1}{2}\log\left(\frac{2\pi}{\beta k}\right) - \frac{3\Gamma}{4\beta k^2}$$

$$\langle E \rangle = -\frac{\partial \log Z}{\partial \beta} = \frac{1}{2\beta} + \frac{3\Gamma}{4\beta^2 k^2} = \boxed{\frac{k_B T}{2} - \frac{3\Gamma}{4}\left(\frac{k_B T}{k}\right)^2}$$

IF $\Gamma = 0$ $\quad \langle E \rangle = \frac{k_B T}{2}$ AS EXPECTED FROM EQUIPARTITION

$$\langle x^2 \rangle = -\frac{2}{\beta}\frac{\partial}{\partial k} \log Z = -\frac{2}{\beta}\left[-\frac{1}{2k} + \frac{3\Gamma \cdot \cancel{2}}{\cancel{4}_2 \beta k^3}\right] = \boxed{\frac{k_B T}{k} - \frac{3\Gamma (k_B T)^2}{k^3}}$$

$$\langle x^4 \rangle = -\frac{4}{\beta}\frac{\partial}{\partial \Gamma} \log Z = \frac{\cancel{4}}{\beta}\frac{3}{\cancel{4}\beta k^2} = \boxed{\frac{3(k_B T)^2}{k^2}}$$

if $\phi(x) = \frac{k x^2}{2} + \frac{\Gamma x^4}{4}$ $\qquad \langle x \frac{\partial \phi}{\partial x} \rangle = k_B T$

$$\frac{\partial \phi}{\partial x} = kx + \Gamma x^3 \qquad \langle x \frac{\partial \phi}{\partial x} \rangle = \langle k x^2 \rangle + \langle \Gamma x^4 \rangle$$

$$\langle x \frac{\partial \phi}{\partial x} \rangle = k\left[\frac{k_B T}{k} - \cancel{\frac{3\Gamma (k_B T)^2}{k^3}}\right] + \Gamma \cancel{\frac{3(k_B T)^2}{k^2}} = k_B T$$

GAS IN A TWO DIMENSIONAL DISK

PARTITION FUNCTION OF SINGLE PARTICLE

$$Z_1 = \frac{1}{\lambda_T^2} \int_{\text{DISK}} dq_x dq_y \, e^{-\beta \frac{k}{2}(q_x^2 + q_y^2)} \qquad r \equiv \sqrt{q_x^2 + q_y^2}$$

$$= \frac{1}{\lambda_T^2} \int_0^R 2\pi r dr \, e^{-\beta \frac{k}{2} r^2} \qquad x \equiv r^2 \qquad dx = 2r dr$$

$$= \frac{\pi}{\lambda_T^2} \int_0^{R^2} dx \, e^{-\beta k \frac{x}{2}} = \frac{\pi}{\lambda_T^2} \frac{2}{\beta k}\left(1 - e^{-\beta k \frac{R^2}{2}}\right)$$

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$$\langle E \rangle = -N\frac{\partial \log Z_1}{\partial \beta} = N\frac{\partial}{\partial \beta}\Big[2\log \underbrace{\lambda_T}_{\lambda_T \sim \beta^{1/2}} + \log\beta + \log\left(1-e^{-\beta\frac{kR^2}{2}}\right)\Big] =$$

$$\boxed{= 2Nk_BT - N\frac{e^{-\beta\frac{kR^2}{2}}}{1-e^{-\beta\frac{kR^2}{2}}}}$$

$$C_V = \frac{\partial E}{\partial T} = 2Nk_B - N\frac{\partial\beta}{\partial T}\frac{\partial}{\partial\beta}\frac{1}{e^{\beta\frac{kR^2}{2}}-1}$$

$$= 2Nk_B + \frac{N}{k_BT^2}\frac{-e^{\beta\frac{kR^2}{2}}}{\left(e^{\beta\frac{kR^2}{2}}-1\right)^2}\frac{kR^2}{2}$$

$$P = -\frac{\partial F}{\partial V}\Big|_{N,T} = Nk_BT\frac{\partial \log Z_1}{\partial V} \qquad V = \pi R^2$$

$$dV = 2\pi R\,dR$$

$$= \frac{Nk_BT}{2\pi R}\frac{\partial}{\partial R}\log\left(1-e^{-\beta\frac{kR^2}{2}}\right) =$$

$$= \frac{N\cancel{k_BT}}{2\pi\cancel{R}}\frac{e^{-\beta\frac{kR^2}{2}}}{1-e^{-\beta\frac{kR^2}{2}}}\cancel{\beta}k\cancel{R} = \boxed{\frac{Nk}{2\pi}\frac{1}{e^{\beta\frac{kR^2}{2}}-1}}$$

Note that as $k \to 0$ $\quad P \approx \frac{N\cancel{k}}{\cancel{2}\pi}\frac{1}{\beta\cancel{k}R^2/\cancel{2}} = \frac{Nk_BT}{\pi R^2}$

(IDEAL GAS)

$$P(a<R<b) = \frac{\int_a^b 2\pi r\,dr\, e^{-\beta\frac{kr^2}{2}}}{\int_0^R 2\pi r\,dr\, e^{-\beta\frac{kr^2}{2}}} = \boxed{\frac{e^{-\beta\frac{ka^2}{2}} - e^{-\beta\frac{kb^2}{2}}}{1-e^{-\beta\frac{kR^2}{2}}}}$$

HARD RODS

TOTAL PARTITION FUNCTION $\quad Z = \dfrac{(L-N\sigma)^N}{N!\,\lambda_T^N}$

ONLY TEMPERATURE DEPENDENCE IS HERE

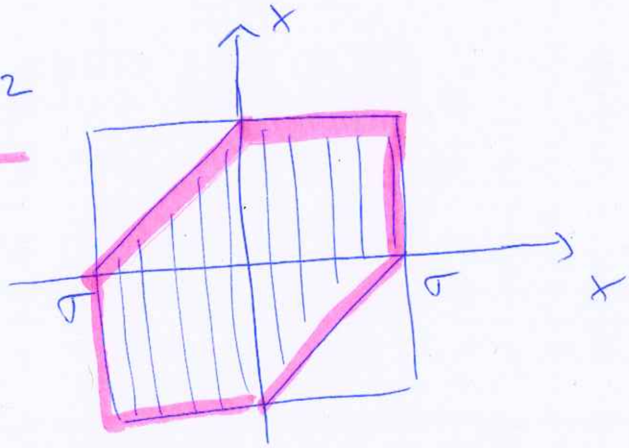
$$\langle E \rangle = -\frac{\partial \log Z}{\partial \beta} = \boxed{\frac{N k_B T}{2}}$$

$$P = -\frac{\partial F}{\partial L} = +k_B T \frac{\partial \log Z}{\partial L} = \boxed{\frac{N k_B T}{L - N\sigma}}$$

$$b_2 = -\frac{1}{2}\int dx \left(e^{-\beta\varphi(|x|)} - 1\right) = -\frac{1}{2}\int_{-\sigma}^{\sigma} dx\,(-1) = \sigma$$

$$b_3 = -\frac{1}{3}\int dx\,dx'\; f(|x|)\, f(|x'|)\, f(|x-x'|) =$$

$$= +\frac{1}{3}\int_{\substack{|x|<\sigma \\ |x'|<\sigma \\ |x-x'|<\sigma}} dx\,dx' = +\frac{1}{\cancel{3}}\cancel{3}\sigma^2$$



$$P = \frac{N k_B T}{L}\,\frac{1}{1-\frac{N}{L}\sigma} \approx \frac{N k_B T}{L}\left(1 + n\sigma + n^2\sigma^2 + \dots\right)$$

$$n = \frac{N}{L}$$