Stat mech examen 2025

January 2025

1 Theory

1.1 Equipartition Theorem

Prove the equipartition theorem. Link it to the following statement: each quadratic degree of freedom contributes $kT/2$ to the total energy. Explain why it doesn't hold for low temperatures.

1.2 Blackbody

Derive the energy density for an ideal blackbody. What is the dependence of the total energy on T? Compare these results to those from classical statistical mechanics.

2 Exercises

2.1 Van der Waals [2pts]

Given the Van der Waals equation find the critical points and critical exponents. Find the relation between a small variation in p and one in V.

2.2 Bosons [3pts]

An average number N of bosons of spin $S = 0$ is confined to a two-dimensional domain with surface A. The gas is ultrarelativistic with a single particle energy $\epsilon = cp$, where c is the speed of light and p is the absolute value of the momentum.
a) Define $z = e^{\beta\mu}$, with μ the chemical potential and $\beta = 1/kT$. Compute N as a function of z. Assume that the system is at a high temperature T. Your answer should give $N(z, A, T)$ and you should expand it up to terms quadratic in z.
b) Compute the pressure, P, of this system as a function of z, A and T. While still being in the high temperature regime, use the result for $N(z, A, T)$ to find $P(N, A, T)$ (keep up to the quadratic terms in N). Discuss your results and the relation to the ideal gas law.

2.3 Quantum system [2pts]

Consider a system of three quantum particles. Each of them can be in one of three states with energies $0, 2\epsilon$ and 5ϵ. The particles are in equilibrium with temperature T.
a) Compute the canonical partition function and internal energy of the system assuming that the particles obey the Bose-Einstein statistics. Discuss the low and high temperature limits.
b) Now assume that the particles obey Fermi-Dirac statistics. What is the canonical partition function and the internal energy? Discuss again the low and high temperature limits.

2.4 Classical molecules [3 pts]

Consider N diatomic molecules with $V = b|r_1 - r_2|^2$ in 3 dimensions. Find the partition function, pressure and energy of the system.