Stat mech exam 2025 V2

Lowwwww taper fade *

29 January 2025

1 Theory

1.1 CSM

In the thermodynamic limit we get the same results for thermodynamic quantities independent of the ensemble we choose. To illustrate this derive the energy $E(N, V, T)$ for an ideal gas in 2 different ensembles. Also discuss the fluctuations of the energy in both of these ensembles.

1.2 QSM

In QSM we have

$$\log \Xi = \mp \sum_{\gamma} \log(1 \mp e^{\beta(\mu - \epsilon_\gamma)}), \tag{1}$$

where Ξ is the grand canonical partition function. Derive this equation and explain your steps. Discuss where quantum mechanics is relevant both in your derivation and the end result. Find the formula for the average occupation number of a state.

2 Exercises

2.1 Relativistic gas

Consider a relativistic gas at temperature T, volume V and N particles with $H = |\vec{p}|c$. Find the partition function. Find the total energy for the system, and discuss how this agrees with equipartition. Also find the pressure of this system.

*Still massive

2.2 Chain of oscillators

Consider a chain of harmonic oscillators where

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \sum_{i=1}^{N-1} \frac{K}{2}(x_{i+1} - x_i)^2. \tag{2}$$

Find the expected value of energy and its variance, and discuss them in the thermodynamic limit. Find the expected value of $(x_1 - x_N)^2$.

2.3 Quantum partition function

Consider a quantum system with $E_n = n\epsilon$ where each state is n times degenerate. [1] Find the partition function for this system and the expected value of energy. Discuss the high and low temperature limits for temperature of the energy.

2.4 Debye model

Atoms in a solid vibrate about their respective equilibrium positions with small amplitudes. Debye approximated the normal vibrations with the elastic vibrations of an isotropic continuous body and assumed that the number of vibrational modes $g(\omega)d\omega$ having angular frequencies between ω and $\omega + d\omega$ is given by:

$$g(\omega) = V\frac{\omega^2}{2\pi^2}\left(\frac{1}{c_L^3} + \frac{2}{c_T^3}\right), \quad \omega < \omega_D \tag{3}$$

where c_L and c_T denote the velocities of longitudinal and transverse waves, respectively. The Debye frequency ω_D is determined by:

$$\int_0^{\omega_D} g(\omega)d\omega = 3N \tag{4}$$

where N is the number of atoms, and hence $3N$ is the number of degrees of freedom.

(a) Calculate the specific heat at constant volume with this model.

(b) Examine its temperature dependence at high as well as low temperatures.

[1] Slightly rephrased from the exam, to be clear $n = 0$ doesn't occur.