Examen Stochastische processen
23 augustus 2018 NM

Naam:..................

- Schrijf je antwoorden op genummerde pagina's. Schrijf je naam op elke bladzijde en start een nieuwe pagina bij elke vraag. Kladwerk dien je ook in, maar apart.

- Het examen is schriftelijk met open boek (zonder boeken).

1. Consider the Markov diffusion process for a position $x_t \in \mathbb{R}$,

$$\dot{x}_t = -\alpha\, x_t + 2\,\xi_t$$

where ξ_t is white noise and $\alpha > 0$ is some parameter.
At time two $(t = 2)$ we have $x_{t=2} = 3$. Find the time-correlation function $\langle x_t\, x_s \rangle$ for all times $3 \leq t \leq s$.

2. Consider a collection of spins, each having two possible values, $\sigma_i = \pm 1$ for $i = 1, \ldots, N$.
a) How many different values are possible for the magnetization $m_N = \sum_{i=1}^{N} \sigma_i$?
b) We condider now a dynamics where a spin σ_j gets flipped with transition rate $\exp(-\beta\, \sigma_j m_N)$, for all j. Describe the evolution of m_N as a continuous time Markov process by giving the backward generator.
c) What is the stationary distribution?

3. Let $\lambda, \mu > 0$ and consider the Markov process on $\{1, 2\}$ with generator

$$L = \begin{pmatrix} -\mu & \mu \\ \lambda & -\lambda \end{pmatrix}$$

a) Calculate L^n and sum $\sum_{n=0}^{\infty} t^n/n! \, L^n$. Compare your answer with the matrix $\exp tL$.

b) Solve the equation $\rho L = 0$, to find the stationary distribution. Verify that $p_t(x, y) \to \rho(y)$ as $t \uparrow +\infty$.

4. Consider a random walker on a ring with N sites in continuous time. The rate to move one step to the right (clockwise) is p, and the rate to move one step to the left (counter clockwise) is q. Suppose that $p/q = e^E$ where $E \geq 0$ is a parameter (external driving field) and that $p + q = \psi(E) > 0$ is a positive function of E.

Compute the clockwise stationary current $j(E)$ as a function of E (and also possibly via the function ψ).

How should we choose the function ψ so that we get negative differential conductivity for large E, i.e., so that

$$\frac{dj}{dE} < 0$$

for large E. You can give an example that works.

5. Explain why detailed balance is related to or is even identical with time-reversal symmetry. Try it in words, with formulae, with examples, with a theorem, a proof, an illustration,... whatever you judge is useful.