Examen Stochastische processen
26 Januari 2018 NM

Naam:.....

- Schrijf je antwoorden op genummerde pagina's. Schrijf je naam op elke bladzijde en start een nieuwe pagina bij elke vraag. Kladwerk dien je ook in, maar apart.
- Het examen is schriftelijk met open boek (zonder boeken).

1. Consider the Markov diffusion process for a position $x_t \in \mathbb{R}$,

$$\dot{x}_t = -U'(x_t) + \sqrt{2T}\xi_t$$

where ξ_t is white noise, $T > 0$ and $U(x) = x^2/2$. At time zero we have $x_0 = 1$.
Find the time-correlation function $\langle x_t x_s \rangle$ for all $0 \leq t \leq s$.
What is the stationary distribution?

2. a) Show that the Ehrenfest model satisfies detailed balance, and find the potential.
b) Show that all Markov chains with two states, $|K| = 2$, satisfy detailed balance, at least when the $p(x, y) > 0$.

3. At time zero a Poisson process $N(t)$ is started with rate μ; $N(0) = 0$. Suppose that (independently of $N(t)$) $X(t)$ is a two-level Markov process, $X(t) \in \{0, 1\}$, with rates $k(0, 1) = a, k(1, 0) = b$, and started from $X(0) = 1$. What is the probability that $X(t) = 1$ during the whole time-period where $N(t) = 1$?

4. Consider the following continuous time Markov process. The state space is $K = \{0, +2, -2\}$ and the transition rates are $k(0, +2) = \exp[-b], k(0, -2) = \exp[-a], k(-2, +2) = k(+2, -2) = 0, k(+2, 0) = \exp[b - h], k(-2, 0) = \exp[a + h]$
Determine the stationary distribution. That asks for the time-invariant state occupation. Is there detailed balance (or, for what values of the parameters a, b, h)?

5. Show that for all observables f,

$$L(f^2) \geq 2\, f\, Lf$$

for the generator L of a Markov process.