Examen Stochastische processen
27 Januari 2017 NM

Naam:.....

- Schrijf je antwoorden op genummerde pagina's. Schrijf je naam op elke bladzijde en start een nieuwe pagina bij elke vraag. Kladwerk dien je ook in, maar apart.
- Het examen is schriftelijk met open boek (zonder boeken).

1. Imagine a continuous time Markov process with a finite state space with (backward) generator L.
Write ρ for the stationary distribution and suppose that $\rho(x) \neq 0$ for all states x. Show that if the matrix H with elements

$$H_{xy} = \sqrt{\rho(x)}\, L_{xy} \frac{1}{\sqrt{\rho(y)}}$$

is symmetric, that then detailed balance holds.

2. Consider a continuous-time Markov jump process on $\{a, b, c\}$ with transition rates $k(a, b) = 1, k(a, c) = k(c, a) = x,\ k(b, c) = 4x/3$ and with all other transition rates equal to zero. Here $x \geq 0$ is a parameter. Give the stationary distribution in terms of x.
Is the stationary process time-reversal invariant — for which x?

3. Consider the Markov diffusion process for a position $x_t \in \mathbb{R}$,

$$\dot{x}_t = -U'(x_t) + \sqrt{2T}\xi_t$$

where ξ_t is white noise, $T > 0$ and $U(x) = x^2/2$. At time zero we have $x_0 = 1$.

Find the time-correlation function $\langle x_t x_s \rangle$.
What is the stationary distribution?

4. Markov found the following empirical rule for the transition matrix in the vowel-consonant space in Pushkin's novel: $\begin{pmatrix} 0.128 & 0.872 \\ 0.663 & 0.337 \end{pmatrix}$. With what vowel versus consonant frequency is that consistent? Give the calculation.

5. Show that the Ehrenfest model satisfies detailed balance, and find the potential.

6. Show that all Markov chains with two states, $|K| = 2$, satisfy detailed balance, at least when the $p(x, y) > 0$.