Exam questions Theoretical part Particle Physics

January 18th, 2016

It is allowed to take $\hbar = c = 1$.

1 A simple toy model for Feynman diagrams

Consider following particle scattering:

$$a + b \to c + d.$$

Draw the diagrams for the two time-ordered processes for this scattering process. Compute for each the matrix element $\mathcal{M}_{fi}$ and show that the sum of both is a Lorentz-invariant expression. Draw then the Feynman diagram for this process. You can take the Lorentz-invariant transition matrix elements at the vertices as constants g.

2 Massive photons

We add a mass contribution to the free Maxwell langrangian density:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{m^2}{2}A^\mu A_\mu. \tag{2.1}$$

- For which values of the mass m is this Lagrangian density gauge invariant?
- Show that the equation of motion from this Lagrangian density is:

$$(\Box + m^2)A^\mu - \partial^\mu(\partial_\nu A^\nu) = 0. \tag{2.2}$$

- When m is non-zero, show that the Lorentz-gauge $\partial_\mu A^\mu = 0$ is a consequence of the equation of motion.
- As in the massless case, we can construct plane wave solutions:

$$A^\mu = \epsilon^\mu \exp(-iqx). \tag{2.3}$$

  Show that there are three independent solutions for the polarisation vector $\epsilon^\mu_{(\lambda)}$ with $\lambda = 1, 2, 3$.

- Prove following completeness relation:

$$\sum_{\lambda=1}^{3} (\epsilon^\mu_{(\lambda)})^* \epsilon^\nu_{(\lambda)} = -\eta^{\mu\nu} + \frac{q^\mu q^\nu}{m^2} \tag{2.4}$$