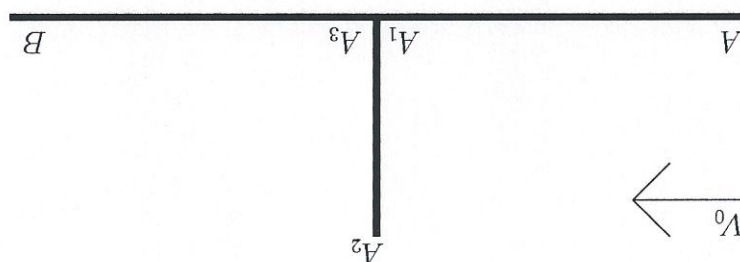


Exercise on Potential Theory

We will search for an equation for the stream lines of a fluid that passes an obstacle in the form of a line of height h .



The fluid enters from the left with a speed V_0 in the horizontal direction when we are far from the obstacle. The plane of the figure will be the complex plane $w = u + iv$.

1. Consider $w(z) = (z^2 - 1)^{1/2}$. What are the properties of this map? Where are the singularities? which type of singularities?

2. Make choices for the latter map such that the function makes a jump when crossing the real line $z = x$ at $-1 < x < 1$. Explain the choices that have to be taken. Draw the z -plane and indicate the value of $w(z)$ for z real, or z close to the real axis, in terms of the positive $\sqrt{x^2 - 1}$ (if $|x| > 1$) or $\sqrt{1 - x^2}$ (if $|x| < 1$).

3. Compare with the figure above. Where are the points A , B , A_1 , A_2 , A_3 in the z plane?

4. Is the map everywhere 'conformal'? or where is it not conformal?

5. Why is the flow in the z -plane described by a velocity vector

$$\vec{V} = \vec{\nabla}\Phi, \quad \Phi = V_0 \operatorname{Re} z.$$

Give all arguments why this is the correct description.

6. Describe in words how you can obtain the velocity vector at each point (u, v) . You do not have to calculate; just tell in words.