Exam General Relativity

17 january 2018

Question 1

The proper time between two time-like separated events is given by

$$\tau_{AB} = \int_A^B d\tau = \int_A^B d\sigma(-g_{\alpha\beta}(x)\frac{dx}{d\sigma}\frac{dx^\beta}{d\sigma})^{1/2} \quad (1)$$

1. State the variational principle for free particle motion in General Relativity.
2. Derive the equations of motion of a free particle.
3. Show that if $g_{\alpha\beta}$ is independent of a coordinate, then there is a conserved quantity associated to this coordinate.

Extra questions: Why do we get a different result in the GR variational principle than in the Newtonian variational principle? Why does curvature of spacetime only work for gravity and not for electromagnetism?

Question 2

Example 9.2 in Hartle (was also an exercise in one of the example classes)
Extra question: Make a drawing and explain what the critical angle is, how we can see it/see what happens.

Question 3

The line element of a Robertson-Walker universe is given by

$$ds^2 = -dt^2 + a(t)^2(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2) \quad (2)$$

where $k = +1, 0, -1$ denotes a resp. closed, open and flat universe. The Friedmann-Lemaître equation is also given:

$$\dot{a}^2 = \frac{8\pi G\rho}{3}a^2 - k \quad (3)$$

1. Show that if a is increasing in time, then an observer will see the light from a distant galaxy redshifted. Derive the Hubble-Lemaître law.

2. Derive an opproximate solution of a(t) from equation (3) in a flat Robertson-Walker universe that has radiation, matter and vacuum energy. Make a sketch of your solution and indicate some important events in the history of our universe.

3. Use equation (3) to derive what will happen in the far future if the vacuum energy density is negative.

Extra question: Why does the energy density of a radiation-dominated universe have a fourth power of a?