Relativity exam 2020

January 2020

1 Part One

The first question is 5 points and is the following:

A Given that for a Killing vector $\Delta_{(\mu}K_{\nu)} = 0$, use this to show that

$$\nabla_\mu \nabla_\nu K^\rho = R^\rho_{\mu\nu\lambda} K^\lambda \tag{1}$$

B Using the following Killing vector $A^\mu = K^\mu$ and the following field $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, proof the following identities:

(a) $g^{\mu\nu} \nabla_\mu F^{\nu\rho} = 0$

(b) $g^{\mu\nu} \nabla_\mu F_{\nu\rho} = 0$

2 Part two

The second question is 15 points. Assume that the Maxwell field $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the only non-trivial source (?) in spacetime with the following equations of motion:

$$g^{\mu\nu} \nabla_\mu F^{\nu\rho} = 0 \tag{2}$$

$$\nabla_{[\mu} F_{\nu\rho]} = 0 \tag{3}$$

and the following metric:

$$ds^2 = -e^{2\alpha(r,t)} dt^2 + e^{2\beta(r,t)} dr^2 + r^2 \big[d\theta^2 + \sin^2\theta \big(d\chi^2 + \sin^2\chi d\phi^2 \big) \big] \tag{4}$$

Lastly, we reduce the vector potential to the following:

$$A_t = f(r,t) \tag{5}$$

$$A_r = A_\chi = A_\theta = A_\chi = 0 \tag{6}$$

1. Solve the equations of motion and find the functions $\alpha(r,t), \beta(r,t)$ and $f(r,t)$. Discuss the time dependence of the functions. Can you exclude a family of parameters by physical reasoning only?

2. Discuss the physical constants in your solution. Study whether this metric has an event horizon. What happens at asymptotical distances?

3. Discuss the symmetries of the metric. Give the timelike Killing vector. He asked something else but I forgot what it was

4. Discuss how you would incorporate the magnetic monopole into the metric