# GR Sessions 10: Cosmology

### Wednesdays December 12, 2012

1. **More De Sitter Space.** Consider the de Sitter metric in flat slicing:

$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2 \ ,$$

where $H$ is a parameter known as Hubble's constant.

   (a) If two points are at an initial distance $d_0$ at time $t = 0$, what is the distance between them at a later time?

   (b) How long does it take for these two particles to start moving away from each other at the speed of light? Call this time interval $t^\star$.

   (c) What will the separation of these two particles be after $t^\star$?

   (d) Write down the geodesic equations for for a massive particle.

   (e) Find the Ricci scalar for this spacetime.

2. **FRW (Carroll 8.1).** Consider an $(N+n+1)-$dimensional spacetime with coordinates $\{t, x^I, y^i\}$ where $I$ goes from 1 to $N$ and $i$ goes from 1 to $n$. Let the metric be

$$ds^2 = -dt^2 + a^2(t)\delta_{IJ}dx^I dx^J + b^2(t)\gamma_{ij}(y)dy^i dy^j \ ,$$

where $\delta_{IJ}$ is the usual Kronecker delta and $\gamma_{ij}(y)$ is the metric on an $n-$dimensional maximally symmetric spatial manifold. Imagine that we normalize the metric $\gamma$ such that the curvature parameter

$$k = \frac{R(\gamma)}{n(n-1)}$$

is either $+1$, 0, or $-1$, where $R(\gamma)$ is the Ricci scalar corresponding to the metric $\gamma_{ij}$.

   (a) Calculate the Ricci tensor for this metric

   (b) Define an energy-momentum tensor in terms of an energ density $\rho$ and pressure in the $x^I$ and $y^i$ directions, $p^{(N)}$ and $p^{(n)}$:

$$\begin{aligned} T_{00} &= \rho \\ T_{IJ} &= a^2 p^{(N)} \delta_{IJ} \\ T_{ij} &= b^2 p^{(n)} \gamma_{ij} \ . \end{aligned}$$

   Plug the metric and $T_{\mu\nu}$ into Einstein's equations to derive Friedmann-like equations for $a$ and $b$ (three independent equations in all).

   (c) Derive the equations for the energy density and the two pressures at a static solution where $\dot{a} = \dot{b} = \ddot{a} = \ddot{b} = 0$, in terms of $k$, $n$, and $N$. Use these to derive expressions for the equation-of-state parameters $w^{(N)} = p^{(N)}/\rho$ and $w^{(n)} = p^{(n)}/\rho$, valid at the static solution.