Symmetries in Quantum Mechanics

Final Exam — Monday January 23, 2012

1. Clebsch-Gordan coefficients: what are they good for?

2. A system of two particles with positions $\vec{r}_1$, $\vec{r}_2$ and intrinsic spins $\vec{S}_1$, $\vec{S}_2$ is described by the Hamiltonian $H = \frac{\vec{p}_1^2}{2m_1} + \frac{\vec{p}_2^2}{2m_2} + V$. For each of the following choices of interaction potential V, which of the following observables are conserved: (i) momentum, (ii) angular momentum, (iii) orbital angular momentum, (iv) parity:

   (a) $V = |\vec{r}_1|^4 + |\vec{r}_2|^4$

   (b) $V = |\vec{r}_1 - \vec{r}_2|^4$

   (c) $V = |\vec{r}_1 + \vec{r}_2|^4$

   (d) $V = x_1 + x_2 + a\,\vec{S}_1 \cdot \vec{S}_2$

   (e) $V = a/|\vec{r}_1 - \vec{r}_2| + b\,\vec{L}_1 \cdot \vec{S}_1 + c\,\vec{L}_2 \cdot \vec{S}_2 + d\,\vec{S}_1 \cdot \vec{S}_2$.

   Consider both the total quantities and the quantities for the individual particles.

3. Let $|njm\rangle$ be the energy eigenstates of a particle in some spherically symmetric potential, with j, m the usual angular momentum quantum numbers. A perturbation $H \to H + \epsilon(t)\,W$ is applied, which will cause transitions between these states. Give as many selection rules as you can for the first order transition matrix element $\langle n'j'm'|W|njm\rangle$ when (i) $W = e^{-r^2}$, (ii) $W = (x^2 - y^2)\,e^{-r^2}$, (iii) $W = L_x$.

4. A system consists of 100 spin 2 particles. Construct a state with total spin quantum numbers $(J, M) = (200, 199)$.

5. Just by counting degeneracies, find the total angular momentum of the ground state of eight noninteracting identical spin 1/2 particles in a harmonic oscillator potential $V(x, y, z) = \frac{m\omega^2}{2}(x^2 + y^2 + z^2)$ (take into account the Pauli exclusion principle).

6. A system with rotational and time reversal invariance whose energy levels are nondegenerate, except for the degeneracies implied by rotational invariance, cannot have a permanent electric dipole moment in any energy eigenstate. Show this by combining time reversal invariance with the Wigner-Eckart theorem (to relate dipole and angular momentum matrix elements).