GR Sessions 8: More Black holes

Wednesdays November 28, 2012

1. **Schwarzschild Thermodynamics.**

   (a) Consider the Schwarzschild solution for a black hole of mass M. Find the *near horizon metric*, that is the metric as viewed parametrically close to the horizon.

   (b) Show by a coordinate transformation that this metric is that of a constantly accelerator observer in minkowski space (also known as Rindler space) with metric given by

   $$ds^2 = e^{2a\xi}\left(-dt^2 + d\xi^2\right) + \text{ angular },$$

   where a is the acceleration of the observer.

   (c) In the Minkowski vacuum, inertial observers see no particles. Rindler observers (those with constant acceleration), however, see a thermal spectrum at temperature $T = a/2\pi$. Argue using the equivalence principle that the near horizon geometry found above must also exhibit a thermal spectrum and find the temperature of the black hole in terms of M.

   (d) Given the first law of thermodynamics, which relates entropy, energy and temperature via the equation

   $$dE = TdS ,$$

   find the entropy of the black hole. Express the entropy in terms of the area of the black hole horizon.

2. **The many horizons of Schwarzschild-de Sitter** The metric of Schwarzschild-de Sitter is given by

   $$ds^2 = -\left(1 - \frac{2M}{r} - \frac{r^2}{\ell^2}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r} - \frac{r^2}{\ell^2}\right)} + r^2\, d\Omega^2 ,$$

   where the de Sitter length $\ell \equiv \sqrt{3/\Lambda}$ and Λ is the cosmological constant.

   (a) Find the horizons of this metric. One is known as the cosmological horizon and the other is the black hole horizon and the final one is physically irrelevant. *Hint: This problem involves solving a cubic equation, which is quite difficult in general. However if you can get your cubic equation in the form:*

   $$t^3 + pt + q = 0 ,$$

   then you can replace $t = u\cos\theta$ *and by a careful choice of* u*, use the identity* $4\cos^3\theta - 3\cos\theta - \cos(3\theta) = 0$ *to identify the 3 roots. What conditions must be satisified so that all roots are real?*

   (b) Now consider that the black hole has left the de Sitter hoirzon such that we remain with the vacuum de Sitter space

   $$ds^2 = -\left(1 - \frac{r^2}{\ell^2}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{r^2}{\ell^2}\right)} + r^2\, d\Omega^2 .$$

   Compute the area of the de Sitter horizon and compare it with the sum of the two areas of the Schwarzschild de Sitter solution. What does this say about the stability of SdS?