K.U.Leuven **1^de examenperiode 2020-2021**

EXAMEN

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1. (Chapter: Dielectrics and Ferroelectrics) The response of a material to electrical fields may be treated using a classical approach. In this approach the Newton equation is applied to a collection of identical bound particles (e.g. bound to specific lattice sites in a crystal) within the material, each one having mass m and charge q (e.g. this can be applied to the electronic polarizability as well as to the ionic polarizability). Assume there are n of this type of particles per unit volume in the solid. Assume the particles respond to a local electric field of angular frequency ω (i.e. $\boldsymbol{E}_{\boldsymbol{local}}(t) = \boldsymbol{E}_{local}(\omega)\, e^{-i\omega t}$).
   a. Write the Newton equation for the general case of a damped harmonic-oscillator model; i.e. assuming elastic restoring forces (with a spring constant k, corresponding to a resonant frequency ω_0) and a damping term proportional to velocity (e.g. with proportionality constant $m\Gamma$).

   b. Write also the relation between the resonant frequency ω_0 and the spring constant k and the mass of the particle m.

c. Find the complex polarization response, $\boldsymbol{P}(\omega)$, per unit volume, attributed to this type of particles, as a function of the angular frequency. Find an expression for the complex polarizability, $\alpha(\omega)$ of this type of particles (proportionality between polarization and local electric field, i.e. such that $\boldsymbol{P}(\omega) = \epsilon_0 \alpha(\omega)\, \boldsymbol{E}_{local}(\omega)$).

d. Describe qualitatively and sketch how the real and imaginary parts of the polarizability behave around the resonant frequency ω_0. What are the limits for the real and imaginary parts of the polarizability as $\omega \to 0$ and as $\omega \to \infty$?

e. The resonant frequencies of the specific particles are related to the spring constant and the mass of the particles (as written in b). Based on this, which particles have higher resonant frequencies, electrons or ions? Which polarizability response.

2. (Chapter: Plasmons, polarons and polaritons)
   a. Explain briefly what is a phonon-polariton.

   b. Sketch the wave dispersion relation $\omega(k)$ (where ω is the angular frequency and k the wave vector of the wave) for a photon, for a Longitudinal Optical (LO) phonon, for a Transverse Optical (TO) phonon, and for the phonon-polariton. You can assume negligible wavelength dependence for sketching the frequency of the phonon modes, and that the LO phonon mode has an angular frequency ω_{LO} which is higher frequency than the angular frequency of the TO phonon mode (ω_{TO}). Identify in the plot the angular frequency gap for propagation of phonon-polaritons.

3. (Chapter: Dielectrics and Ferroelectrics)
    a. Write the Landau approximation for the free energy of a ferroelectric material as a function of the hypothetical polarization value P, in the absence of an external electric field (ignore the electric field term), and to the 4th power in P (in order to model a second order phase transition).

    b. What (simple) assumptions for the values and temperature dependence of the coefficients of the expansion are appropriate in order to describe a material with a second order ferroelectric phase transition at a temperature T_0?

c. Sketch the free energy dependence on hypothetical P values for $T > T_0$ (paraelectric state), for $T = T_0$ (transition temperature), and for $T < T_0$ (ferroelectric state). Identify in the sketch the stable equilibrium polarizations which can be physically observed in thermodynamic equilibrium. Identify in the plot the unstable equilibrium polarization, which does not correspond to a thermodynamic equilibrium state.

4. (Chapter: Optical Processes and Excitons)
   a. Explain qualitatively what are excitons:

   b. and describe the characteristics of i) Frenkel and ii) Mott-Wannier excitons