Exam Advanced Quantum Mechanics
11 January 2016 PM

Name:..................

- Please write your answers on numbered pages. Write your name on each page. Start a separate page for each new question. Additional pages with your draft work, rough calculations or incomplete answers are handed in separately but are not considered.
- The exam is oral, closed book

1. Consider the coherent states from quantum optics,

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

for $\alpha \in \mathbb{C}$, in terms of the photon number states $|n\rangle$. Show that for the quadrature operators $X = (a + a^*)/2$ and $P = (a - a^*)/(2i)$, their variances in the coherent state $|\alpha\rangle$ equal

$$\langle(\Delta X)^2\rangle_\alpha = \frac{1}{4} = \langle(\Delta P)^2\rangle_\alpha$$

You can use that $a|\alpha\rangle = \alpha|\alpha\rangle$.
To be discussed in the oral part: Is the above formula also valid for the photon number states — for which number states?

2. Oral part: What was the point or the purpose of the Einstein-Podolsky-Rosen paper (1935)?

3. Obtain in the first Born approximation the scattering amplitude, the differential and the total cross-sections for scattering by the Gaussian potential $V(r) = V_0 \exp(-\alpha^2 r^2)$.

Oral part: explain how to obtain the Born approximation from the Lippmann-Schwinger equation

$$\psi_{\vec{k}}(\vec{r}) = e^{ikr} - \frac{1}{4\pi}\int \frac{\exp[ik|\vec{r}-\vec{u}|]}{|\vec{r}-\vec{u}|}\, U(\vec{u})\, \psi_{\vec{k}}(\vec{u})\, \mathrm{d}\vec{u}$$

by solving it iteratively and with the aid of the Green's function

$$G(_0(\vec{k},\vec{r},\vec{u}) = -\frac{1}{4\pi}\, \frac{\exp[ik|\vec{r}-\vec{u}|]}{|\vec{r}-\vec{u}|}$$

For the rest we want to use the expression

$$f(\vec{k},\theta,\varphi) = -\frac{1}{4\pi}\int \exp[-i\vec{k}'\cdot\vec{u}]\, U(\vec{u})\, \psi_{\vec{k}}(\vec{u})\mathrm{d}^3\vec{u} = -\frac{1}{4\pi}\, \langle \phi_{\vec{k}'}|U|\psi_{\vec{k}}\rangle$$

to obtain the scattering amplitude.

4. Apply the gauge transformation generated by taking

$$\chi(\vec{r},t) = -\frac{1}{2} B\, x\, y$$

to the potentials $\vec{A}(\vec{r}) = \frac{1}{2}(\vec{B}\times\vec{r}),\ \phi = 0$, where $\vec{B}$ is taken along the z-axis. Show that the transformed time-independent Schrödinger equation, for a spinless particle of charge $q = -e$ and mass m, is

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + \frac{i\hbar B y}{m}\frac{\partial}{\partial x} + \frac{e^2B^2y^2}{2m}\right)\Phi(\vec{r}) = E\,\Phi(\vec{r})$$

5. Oral part: sketch the argument for obtaining the Aharonov-Bohm effect from path–integral methods.