Exam Advanced Quantum Mechanics 20 January 2017

Name:.....

- Please write your answers on numbered pages. Write your name on each page. Start a separate page for each new question. Additional pages with your draft work, rough calculations or incomplete answers are handed in separately but are not considered.
- The exam is oral, closed book

1. What is the experimental set-up and what are the experimental facts for the Aharanov-Bohm effect applied to the two-split experiment. What changes and how does that change as function of what?

2. Kicked oscillator. Consider the one-dimensional quantum harmonic oscillator in its ground state $|0\rangle$ at very early times, say at time $t = -\infty$. We have for example for the position operator x that

$$\langle 1|x|0\rangle = \sqrt{\hbar/(2m\omega)}$$

with $\hbar \omega$ the constant energy difference between the energy levels and m the mass. Then starts a time-dependent perturbation

$$V(t) = -eE x e^{-t^2/\tau^2}$$

for some time-constant $\tau > 0$. In that way we obtain a time-dependent dynamics. At very late times the state has a component in the first excited state $|1\rangle$ of the harmonic oscillator. Compute the probability to find it there for say $t = +\infty$ to first order in the field E, and see for what τ that probability is maximal. You can take advantage of the identity, for $\lambda, s \in \mathbb{R}$,

$$\int_{-\infty}^{+\infty} \mathrm{d}t' \, \exp[i\lambda t' - t'^2/s^2] = \sqrt{\pi\tau} \exp[-\lambda^2 s^2/4]$$

3. a) What was the purpose of the Einstein-Podolsky-Rosen paper (1935) and what is the (possibly simplified) argument?

b) Explain the difference between superposition and mixture.

c) What is an entangled state or system? Give a specific example to illustrate your point.

4. Obtain in the first Born approximation the scattering amplitude, the differential and the total cross-sections for scattering by the exponential potential $V(r) = V_0 \exp(-\alpha r)$.

5. In what sense is the Jaynes-Cummings model different from the semiclassical treatment of Rabi-oscillations? Different physics – different approximations? What is the physical context?

6. Calculate for a complex number z,

$$e^{za^{*}}e^{-z^{*}a}|0\rangle$$

where, respectively, a and a^* are the annihilation and creation operator for the harmonic oscillator with ground state $|0\rangle$. Call $|n\rangle$ the eigenstate with nparticles/photons.