

**analytical mechanics 25 August 2017 AM**

Name: .....

1. Write down the Lagrangian for the following system: a cart of mass  $m$  can roll without friction on a rail along the  $x$ -axis. A pendulum, consisting of a stick of length  $\ell$  and a point mass  $m$ , is mounted rigidly on the cart and can move freely within the  $x - z$  vertical plane.

2. Give the Liouville equation for the smooth dynamical system  $\dot{x}(t) = f(x(t))$ ,  $x(t) \in \mathbb{R}^n$ .

3. Consider the transformation  $Q = qt$  and  $P = pt$ . Is that transformation canonical?

4. Show that the logistic map for  $r = 4$  is conjugate to the tent map.

5. Show that the periodic points of the Bernoulli shift  $x \mapsto 2x \bmod 1$  on  $[0, 1]$  are dense in  $[0, 1]$ .

6. a) Consider a potential that is a small perturbation of a harmonic potential,

$$V(x) = x^2 + ax^4$$

Calculate the period of the motion up to first order in  $a$ . Make sure to see what steps really need analytic justification.

b) Show that the derivative with respect to the energy of the integral  $\oint p dx$  over one period, equals the period of the motion.