# Exam: Electroweak and Strong Interactions

Master of Physics

28 June 2019

Write your name on each page of your written preparation and include page numbers. We use natural units and set $c = 1$ and $\hbar = 1$ for simplicity.

## 1 Question 1

The Standard Model and a different representation.

For this question consider a different choice for the scalar multiplet. Instead of taking a scalar field in the two-dimensional representation of $SU(2)_L$, we consider a scalar field in the three-dimensional representation. We start with three general complex scalar fields, $\phi_1$, $\phi_2$ and $\phi_3$ which are combined in a $3 \times 1$ array,

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

The Lagrange density, with $m^2, \lambda \in \mathbb{R}$ and strictly positive, is given by:

$$\mathcal{L} = \mathcal{D}_\mu \Phi^\dagger \mathcal{D}^\mu \Phi + m^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2$$

The Lagrange density is invariant under the $SU(2)_L \times U(1)_Y$ transformations:

$$\Phi \to \Phi' = e^{ig\alpha^a t_a} e^{ig' q_Y \beta} \Phi$$

with $\alpha^a, a \in \{1, 2, 3\}$ and $\beta$ are functions and the $t_a$ matrices given by:

$$t_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad t_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \qquad t_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

1. Discuss the structure of the groundstate(s) of the scalar system. [1]
2. Choose the groundstate as

   $$\Phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}$$

   with $v = \sqrt{m^2/\lambda}$ and determine $q_Y$ by requiring that photons are massless.
3. Discuss the physical degrees of freedom and find the unitary gauge of the system.
4. Determine the mass ratio of the $W$ and $Z$ vector bosons and compare this result with what we obtained during the course.
5. Determine whether a Yukawa-type interaction would actually be possible in this representation (think of the direct product of representations in $SU(2)$ and terms in the Lagrangian density have to be singlet representations).

[1] A geometrical interpretation of the solutions can be useful.

## 2 Question 2

In search of the Higgs...

We return to the Standard Model as we developed in quite some detail during the lectures. The coupling of the BEH field ($\sigma$) to leptons and quarks is given by:

$$\mathcal{L}_{HF} = -\frac{1}{v} m_\psi \sigma \overline{\psi} \psi$$

the coupling of the BEH field to the $W$ and $Z$ is given by:

$$\mathcal{L} = \frac{vg^2}{2} \sigma W_\mu^\dagger W^\mu + \frac{g^2}{4} \sigma^2 W_\mu^\dagger W^\mu + \frac{vg^2}{4\cos^2(\theta_W)} \sigma Z_\mu Z^\mu + \frac{g^2}{8\cos^2(\theta_W)} \sigma^2 Z_\mu Z^\mu$$

A few masses in GeV/$c^2$ below:

$$m_W = \frac{vg}{2} \simeq 80$$
$$m_Z = \frac{m_W}{\cos\theta_W} \simeq 91$$

| | | | | | |
|---|---|---|---|---|---|
| $m_e \simeq$ | $0.51 \times 10^{-3}$, | $m_\mu \simeq$ | $0,11$, | $m_\tau \simeq$ | $1,8$, |
| $m_u \simeq$ | $1 - 5 \times 10^{-3}$, | $m_c \simeq$ | $1,15 - 1,35$, | $m_t \simeq$ | $170$, |
| $m_d \simeq$ | $3 - 9 \times 10^{-3}$, | $m_s \simeq$ | $0,075 - 0,17$, | $m_b \simeq$ | $4,0 - 4,4$. |

1. Compare the strength of the Higgs-lepton (and Higgs-quarks) interactions with the corresponding electromagnetic interaction strength (use $g\sin(\theta_W) = e$ given).
2. On the next page there is a figure for the branching ratios of the scalar boson decays as a function of the BEH mass ($m_H = v\sqrt{2\lambda}$). Give a qualitative explanation for relative strength of the branching ratio to $b\overline{b}$ with respect to $\tau\overline{\tau}$ for low BEH masses.
3. Give an explanation with Feynman diagrams of leading order decay of a scalar boson to a gluon-gluon pair and a photon-photon pair.
4. Discuss the production of a $WW$ and $ZZ$ boson pair from the decay of scalar boson and explain when these modes are possible and give the respective decay process. Notice that the first and third term in the Lagrangian density describing these decays are approximately equal in strength, so justify why the $WW$ branching ratio is significantly higher than the $ZZ$ branching ratio.
5. Define the concepts of partial decay width, total decay width and branching ratio of a decay process and explain the dip in the $ZZ$ producing process branching ratio.

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These questions were written from memory and could possibly contain some mistakes, which were of course not present in the original exam.

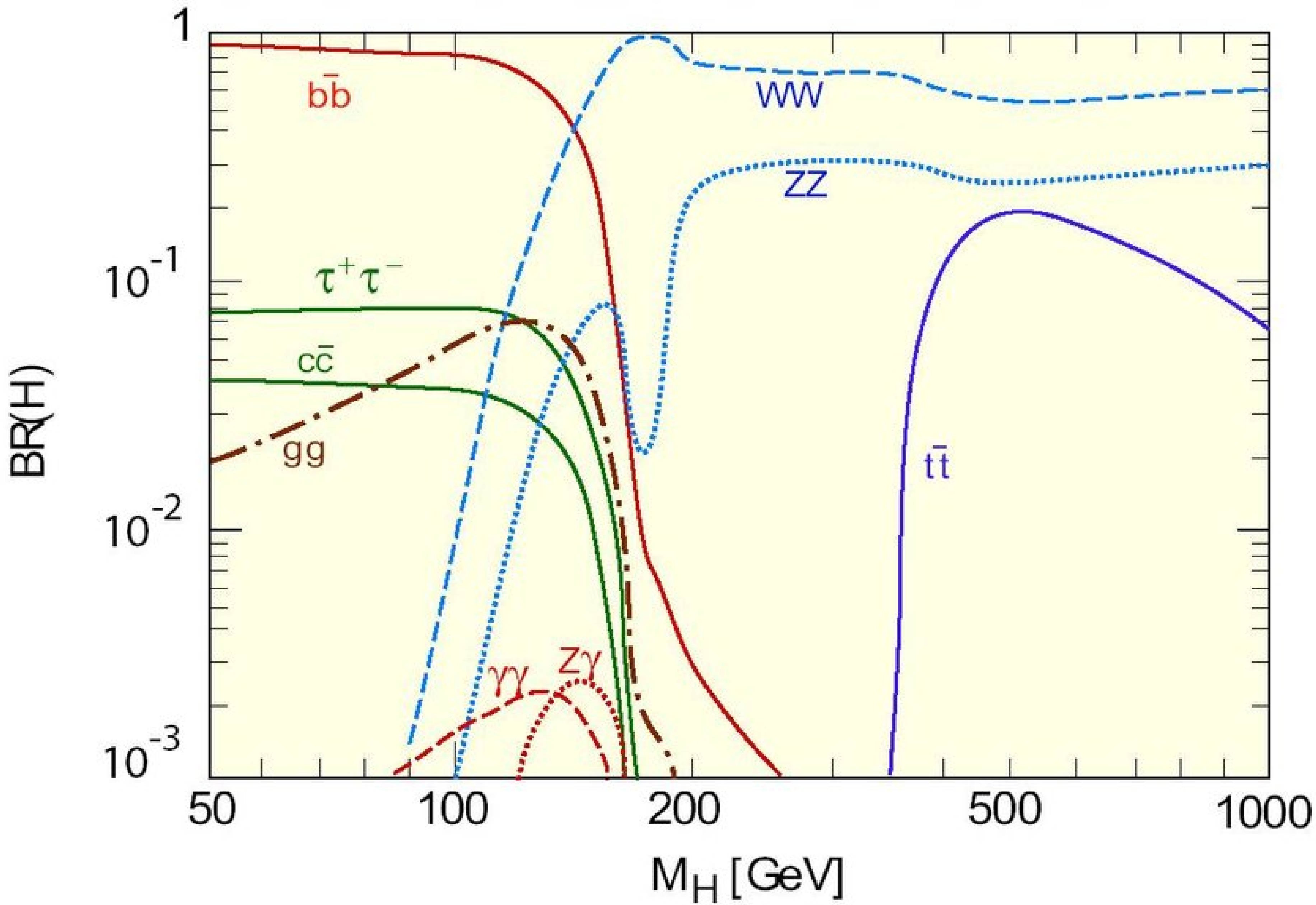


Figure 1: Decay Channels of Higgs Boson in the SM