# Radiative Processes in Astronomy

Friday 19 January 2018. Morning Session.

The exam is divided in 4 questions, where the first one is oral (but where you have the opportunity to prepare a bit in advance). The oral part will begin after 1 hour, so I recommend you start by preparing the discussion of this. You are not allowed to bring any notes to the actual discussion.

For the rest of the exam, you are allowed to use your personal notes from Lectures 1-21, the slides, the course lecture-notes, and a calculator. It is not allowed to use a mobile phone as calculator, nor any other mobile device.

Please write your name on each paper that you use and on the papers with questions. In order to get points on the written questions (2 and 3), you must show clearly your full work. Make sure that your hand-writing is well readable and that you follow a logical structure for your calculations and when answering the questions.
– *If your first solution to a problem looks "messy", I advice you to simply do and hand in a re-writing of your solution using a new, fresh set of papers.*

Good luck !

1. Oral exam. We will discuss some (i.e., perhaps not all) of the following key concepts from the course [7 points] :

   - Radiative diffusion approximation
   - Limb Darkening
   - Lorentz, Doppler and Voigt profiles
   - Eddington-Barbier and formation of a spectral line in LTE
   - Thermal Bremsstrahlung
   - Boltzmann and Saha equations
   - Thermal vs. scaterring source function
   - Compton scattering
   - Solution to transfer equation through homogenous slab
   - Einstein coefficients and relations

2. Moments of the radiation field [5 points] :

   Assume the intensity $I$ can be a function of height $z$ and radiation angle $\mu = \cos\theta$.

   (a) Assume $I$ is given by $I(z, \mu) = I^+(z)$ for $\mu \geq 0$ and $I(z, \mu) = I^-$ for $\mu < 0$. Derive expressions for the 0-2$^{\text{th}}$ angular moments, $J, H, K$, of the intensity. How are these related to the radiation energy density $u$, the radiative flux $F$, and the radiation pressure $P$?

   (b) For the situation in (a) above, what are $K/J$ and $H/J$ at i) the surface $z = 0$ where we have no incoming radiation and ii) deep into the object where the radiation field can be approximated as completely isotropic?

   (c) Assume now that the radiation field deep into the object is not completely isotropic, but nearly so and given by $I(\mu, z) = I_0(z) + \mu I_1(z)$. How does this now change the ratios $K/J$ and $H/J$?

   (d) Assume now instead that $I$ is confined to just two discrete rays in $\mu$, one upward (+) and the other downward (-) with $\mu^- = -\mu^+$, and with associated intensities

   $$I(z, \mu) = I^+(z)\delta(\mu - \mu_0) \tag{1}$$

   for $\mu \geq 0$ and

   $$I(z, \mu) = I^-(z)\delta(\mu + \mu_0) \tag{2}$$

   for $\mu < 0$. Here $\mu_0 > 0$ and $\delta(x - a)$ (=0 for $x \neq a$) is the Dirac-Delta function with property

   $$\int_{a-\epsilon}^{a+\epsilon} f(x)\delta(x - a)dx = f(a) \tag{3}$$

   for all $\epsilon > 0$. For this two-stream approximation, what value of $\mu_0$ should one choose in order to recover the Eddington approximation $K/J = 1/3$? And what is now $H/J$ at the surface $z = 0$ where the incoming radiation is zero?

3. Scattering, radiative acceleration and energy density [5 points] :

(a) For a fully ionized hydrogen (H)+helium (He) plasma with He number density content $Y_{He} = n_{He}/n_H$, derive an expression for the mass absorption coefficient $\kappa_{\rm Th}$ [cm$^2$/g] due to Thomson scattering and compute its numerical value for $Y_{He} = 0.2$.

[Hint: For a helium atom the mass is 4 times higher than that of a hydrogen atom.]

(b) Assuming now that the total radiative acceleration for a spherically symmetric astrophysical object is enhanced with a constant factor $\bar{Q}$ as compared to the radiative acceleration due to Thomson scattering, i.e. that $g_{\rm rad} = \bar{Q} g_{\rm Th}$. Using your value for $\kappa_{\rm Th}$ from (a) above and a fixed total mass $M$, compute the limiting value for $\bar{Q}$ above which the outward pushing $g_{\rm rad}$ exceeds the inward pull of gravitational acceleration for the Sun. Next, assuming now $\bar{Q} = 1$, compute the stellar mass $M_*$ at which the radiative acceleration equals the gravitational acceleration assuming that $L_* \propto M_*^3$ for stellar luminosity $L_*$.

[Note: In case you didn't manage to find a numerical value in (a), simply use a generic value when carrying out the computations in (b).]

(c) Discuss why (for high luminosity objects) spectral line opacity can be a good possibility to obtain a high factor $\bar{Q}$ in (b) above and so significantly boost the radiative acceleration. Motivate your reasoning using the *quality factor* of a resonance for a spectral line viewed as a weakly damped, classical harmonic oscillator. In a scientific application, how would we need to modify this quality factor to account for the effects of quantum mechanics?

(d) Using measurements of the CMB radiation temperature $T_0 = 2.73\,\mathrm{K}$ and mass-density $\rho_0 \approx 5 \times 10^{-31}\,\mathrm{g/cm^3}$ in the present-day Universe, estimate the redshift $z$ at which the radiation energy density $e_R$ and the rest-mass energy density $e_M = \rho c^2$ of the Universe were equal. For this you may assume radiation in thermal equilibrium, mass conservation in a sphere with scale factor $R$, and that the following relations hold: $R \propto (1+z)^{-1}$ and $T \propto 1+z$. Discuss what your calculation tells you about the relative importance of $e_R$ and $e_M$ in the Universe today and in the distant past. [Note: At present time $z = 0$.]

4. Quiz [3 points]:

(a) If you neglect stimulated emission, Einstein's atomic relations are consistent with black-body radiation according to

1) Planck
2) Wien's limit
3) Rayleigh-Jean's limit

(b) The radiation energy density of a black-body depends on its:

1) density
2) temperature
3) density and temperature

(c) For radiation transport through two homogeneous, optically thick slabs, 1 and 2, in LTE with zero incoming intensities, and of temperatures $T_1 > T_2$, the outgoing (emergent) intensities $I_1$ and $I_2$ will have:

1) $I_1 = I_2$
2) $I_1 > I_2$
3) $I_1 < I_2$

(d) For thermal bremsstrahlung with $h\nu/(kT) \ll 1$, the optical depth in a homogeneous medium depends on

1) $\nu^0$
2) $\nu^2$
3) $\nu^{-2}$

(e) Assuming LTE and a pure helium gas, the temperature at which neutral helium ionizes is

1) independent of density
2) higher in a high-density object than in one with lower density
3) higher in a low-density object than in one with higher density

(f) Assuming pure extinction of a light-source, at optical depth 1/3

1) no photons escape
2) between 40-60 % of the emitted photons escape
3) between 60-80 % of the emitted photons escape

(g) The ionization potential of hydrogen (H) from the ground-state is 13.6 eV. This means that

1) The H Lyman $\alpha$ line has a shorter wavelength than 911 Å
2) The H Balmer $\alpha$ line has a shorter wavelength than 911 Å
3) Both the H Balmer and Lyman $\alpha$ lines have longer wavelengths than 911 Å

(h) In local thermodynamic equilibrium

1) $S = B$ but not necessarily $J = B$ or $I = B$
2) $S = B$ and $J = B$ but not necessarily $I = B$
3) $S = B$ and $J = B$ and $I = B$

(i) If the thermal speed of a specific ion is 1 km/s in a $T = 10^2$ K gas, in a $T = 10^4$ K gas it is

1) 10 km/s
2) 100 km/s
3) 1 km/s

(j) The radiative flux of an astrophysical object is radiative

a) energy output
b) energy output per time
c) energy output per time per area

(k) On the HR -diagram, if star A lies at the same place on the vertical axis as star B, but more to the left on the horizontal axis, which star has the largest radius ?

1) A
2) B
3) equal radius

(l) The hydrogen Lyman jump lies

1) at a higher frequency than the hydrogen Balmer jump
2) at a lower frequency than the hydrogen Balmer jump
3) at the same frequency as the hydrogen Balmer jump