Radiative Processes in Astronomy

Monday 14 January 2019. Afternoon Session.

The exam is divided in 4 questions, where the first one is oral (but where you have the opportunity to prepare a bit in advance). The oral part will begin after 1 hour, so I recommend you start by preparing the discussion of this. You are NOT allowed to bring any notes to the actual discussion with me.

For the rest of the exam, you are allowed to use the course lecture-notes, and a calculator. It is NOT allowed to use a mobile phone as calculator, nor any other mobile device. It is also NOT allowed to use any notes from solutions to exercises done in class or by yourself, nor any of the pdf-slide lectures.

Please write your name on each paper that you use and on the papers with questions. In order to get points on the written questions (2 and 3), you must show clearly your full work. Make sure that your hand-writing is well readable and that you follow a logical structure for your calculations and when answering the questions.

– If your first solution to a problem looks ”messy”, I advice you to simply do and hand in a re-writing of your solution using a new, fresh set of papers.

Good luck !

1. Oral exam. We will discuss some (i.e., perhaps not all) of the following key concepts from the course [7 points] :
    - Elastic scattering as a classical oscillator (including the three regimes)
    - Radiative diffusion approximation
    - Wien's displacement law
    - Thermal Bremsstrahlung
    - Atomic transitions of hydrogen (physics origin, spectral domains, astrophysical applications)
    - Angular moments of the intensity and corresponding physical quantities
    - Limb Darkening
    - Radiative acceleration and the Eddington limit

2. Line formation and line broadening [6 points] :

   Approximate an astrophysical gas cloud as a homogeneous, plane-parallel slab in LTE and of a constant temperature T. The slab has an incident radiation I_0 entering from below, which you may take to be the same within the line l and in the nearby continuum c. When viewing the slab from above, what will you observe?

   (a) Assume first that you can neglect extinction in the continuum, such that $\alpha_l > \alpha_c = 0$ and $\tau_l > \tau_c = 0$. Now for the case of $0 < \tau_l << 1$, show under which conditions you will observe i) emission lines and ii) absorption lines superimposed upon the nearby continuum I_0. Discuss astrophysical examples.

   (b) Now for the case of $\tau_l >> 1$ and also $\tau_c >> 1$, will you see any absorption or emission lines? What type of spectrum will you observe?

   (c) Assume now a neutral pure hydrogen gas of $T = 200$ K and that thermal Doppler broadening dominates the line profile. Compute the full-width-half-maximum (FWHM) of the line profile at a typical frequency $\nu = 3 \times 10^{15}$ Hz.

   (d) For the situation in (c), what mass density (in g/cm^3 or kg/m^3) must the cloud have in order for collisional line broadening to be equally important as Doppler line broadening at FWHM?

   HINTS: For a Lorenz profile, FWHM occurs at $\gamma/(2\pi)$ and for collisional line broadening $\gamma = 2/t$. For the mean time between collisions you can here assume $t = \ell/v_{th}$ and to compute the collisional mean-free-path ℓ you can simply assume the geometrical cross-section of a Bohr atom.

   NOTE: Partial credit will be given, so if you are not able to find reasonable numerical values to (c) and (d), still provide your full calculations and a discussion about what you think might have gone wrong.

   MATH: Taylor expansion of e^{-x} for $x << 1$ is $e^{-x} \approx 1 - x$.

3. Saha-Boltzmann and ionization states [3 points] :

   (a) For a pure hydrogen gas in LTE with mass density $\rho = 10^{-9}\,\mathrm{g/cm^3}$ and temperature $T = 5800$ K, compute the fraction of ionized to total number of hydrogen atoms. Are the hydrogen atoms mostly neutral or ionized? What astrophysical object might this be a reasonable approximation for?

   NOTE: For the hydrogen partition functions, you can assume $U_{HII}/U_{HI} = 1/2$.

   (b) In Ch. 8.1 in the lecture-notes, we compute the ionization state of a hydrogen Universe as a whole, and find that at $T = 5800$ K the Universe is almost fully ionized. Does this result differ from the one you have found in (a) above? If so, why? And what does this tell you about the ionization states in different astrophysical situations?

   MATH: Solution to equation $ax^2 + bx + c = 0$ is
   $x = (-b \pm \sqrt{b^2 - 4ac})/(2a)$.

4. Quiz [4 points]:

   Correct answer: 0.25 points/question. Incorrect answer: 0 points/question.

   (a) The mean free path of photons in a homogeneous, grey medium is reached when

   1) $\tau = 2/3$
   2) $\tau = 1$
   3) It's never reached

   (b) The radiation pressure of a black-body depends on its:

   1) density
   2) temperature
   3) density and temperature

   (c) For thermal bremsstrahlung with $h\nu/(kT) \ll 1$, the optical depth in a homogeneous medium depends on

   1) ν^0
   2) ν^2
   3) ν^{-2}

   (d) For a fully ionized gas, the mass absorption coefficient κ (cm^2/g) of Thomson scattering is

   1) equal in a pure hydrogen and pure helium gas
   2) higher in pure helium gas than in a pure hydrogen gas
   3) lower in a pure helium gas than in a pure hydrogen gas

   (e) You are computing the transfer of radiation through a layer δR within a spherically symmetric object of radius R. The plane-parallel approximation is

   1) valid if $\delta r \ll R$
   2) valid if $\delta r \approx R$
   3) always valid

   (f) Scattering of photons generally tend to drive a gas

   1) into TE
   2) out of TE but into LTE
   3) out of TE and out of LTE

(g) Assuming pure extinction of a light-source, at optical depth 2

1) between 1-2 % of the emitted photons escape
2) between 10-20 % of the emitted photons escape
3) no photons escape

(h) The mass-density goes from being highest to lowest in the following astrophysical media

1) stellar atmosphere, star's centre, the interstellar medium (ISM)
2) star's centre, stellar atmosphere, the ISM
3) the ISM, stellar atmosphere, star's centre

(i) On the HR -diagram, if a star lies higher on the vertical axis than the Sun, and more to the right on the horizontal axis

1) Its radius is larger than the Sun's and its T_{eff} higher
2) Its radius is larger than the Sun's and its T_{eff} lower
3) Its radius is smaller than the Sun's and its T_{eff} lower

(j) When deriving the Eddington-Barbier relation for the emergent intensity, we assume

1) an optically thick medium
2) a source function that depends linearly on optical depth
3) both of the above

(k) Above Eddington's upper mass limit for stars, the magnitude of the radiative acceleration g_{rad} is

1) lower than the gravitational acceleration
2) higher than the gravitational acceleration
3) unrelated to the gravitational acceleration

(l) The ionization limit from CI in the ground state is 11.3 eV; all CI $n = 2 \rightarrow 3$ transitions correspond to energies

1) $= 11.3eV$
2) $< 11.3eV$
3) $> 11.3eV$

(m) In the case of an isotropic radiation field, the ration J/K is:

1) 1/3
2) 1
3) 3

(n) Is it correct to describe Compton scattering as a special case of a more general Thomson scattering case?

1) Yes
2) No
3) They are always exactly the same thing

(o) The effective temperature of an astrophysical object is really a proxy-quantity for its

1) intensity
2) radiative flux
3) energy density

(p) For a black-body radiator with $T = 2K$, we can apply the Rayleigh-Jeans approximation when analyzing its infrared radiation

1) Yes, if $\rho > 10^{-9}$ g/cm^3
2) No
3) Yes