Exam Groups and Symmetries

student

January 2021

Theory

You have 30 minutes for this part.

1. Let $\mathfrak{g}$ be a Lie algebra. Define $\mathfrak{h} = [\mathfrak{g}, \mathfrak{g}]$. What is the name of $\mathfrak{h}$ and is it an ideal?
2. What are positive roots? Define simple roots.
3. How many fundamental weights are there? Define them.
4. What are the quaternion matrices used for, in the context of real lie algebras? Choose one method and briefly explain.
5. Compare $SU(2)$ and $SO(3)$. Connected, simply connected, covering group, representations.
6. What is a basic module? Is the tensor product of two basic modules irreducible?
7. Explain the difference between regular and special subalgebras. Give an example of both for $\mathfrak{su}(3)$.
8. For $\mathfrak{sl}(4)$ the basic representation is $\square$. What is the conjugate representation?

Exercises

Due to the measures surrounding Covid-19, you had 2,5 hours for this.

1

Basically do everything we have done in the course for the lie algebra $Sp(4)$. You were given the basis that was also in the book for C_2.

1. Check that the basis elements of the CSA are in $Sp(4)$.

2. Check that the basis step operators are in $Sp(4)$.
3. Calculate the commutation relations between these.
4. Defining the following 4 roots as positive, which ones are the simple roots? Name them as was done in the course.

$$(1,-1),(2,0),(1,1),(0,2)$$

5. What is the height of each root? What is the highest root? How do you know?
6. Use the definition of H^i, to find H^1 and H^2 as linear combination of the first basis of the CSA.
7. Give the cartan matrix.
8. Give G_{ij} by considering the inner product of H^i and H^j. Check that it relates to A^{ij} as it should.
9. Calculate G^{ij} and the fundamental weights.
10. Draw a diagram with the roots and the fundamental weights.

2

Derive from the definition the character formule of a Verma module of A_1 in function of the highest weight Λ. Recall how we did this for the finite dimensional highest weight representation of A_1, find this result via your earlier derived formula.