Theory

Question 1 (written)
Show that pushouts in a category $\mathcal{C}$ are unique (up to equivalence), if they exist.
Question 2 (oral)
Questions on the Basis Theorem 9.12 and its "sophisticated proof" in Rotman's book.
Question 3 (oral)
Surprise Question.

Exercises

Question 1 (related to the assignments)
(a) (Exercise 1 of assignment 2) Does such a category $\mathcal{G}$ have pullbacks?
(b) (Exercise 2 of assignment 2)

1. We say that a ring is *reduced* if its nilradical is trivial (i.e. $\text{Nilrad}(R) = \{0\}$). State and prove that reducedness is a local-global property.

2. Is the property *flatness* a local-global property?

Question 2
Write the following Aberlian group as a product of cyclic subgroups and find a generator for each subgroup: the group G with generators a, b, c, d, e and f and relations $5a + 10d = 2b + 4c + 2d + 4f = 5a + c + 10d + 5e = 5a + 3c + 15d + 3e + 2f = 0$
Question 3
Consider the ring $R = \mathbb{C}[x^2, x^3]$ with maximal ideal $m = (x^2, x^3)$ and the ring $S = \mathbb{C}[X]$ with maximal ideal $n = (x)$. Since R is a subring of S, we can view n as an R-module, hence $M = R/m \otimes_R n$ is a module over $R/m \cong \mathbb{C}$. Note that every module over a field is always free. Compute the rank of M.
Question 4
(a) Let R be a ring and

$$0 \to F_2 \to F_1 \to F_0 \to 0$$

a short exact sequence of finitely generated free R-modules. Show that:

$$\text{Rank}(F_1) = \text{Rank}(F_2) + \text{Rank}(F_0)$$

(b) Show that any finitely generated projective module over a local ring is free.
Hint: use Nakayama's lemma (exercise 2.10) and recall that the jacobson radical of a ring is the intersection of all maximal ideals.
(c) Let R be a local ring and let

$$0 \to F_n \to F_{n-1} \to ... \to F_0 \to 0$$

be an exact sequence of finitely generated free R-modules. Show that

$$\sum_{i=0}^{\infty} (-1)^i \text{Rank}(F_i) = 0$$