Electrodynamica — 7/9/2012

Om ambiguteit in terminologie te vermijden zijn de vragen in het Engels opgesteld. Als je bepaalde woorden of vragen in het Nederlands wilt vertaald krijgen kan je dit om het even wanneer komen vragen. Het examen is mondeling met schriftelijke voorbereiding.

1. Give a brief answer to the following questions and explain qualitatively yet clearly and precisely why your answer is correct:

   (a) Why do pretty much all commercially available capacitors have highly polarizable dielectrics in them?

   (b) Explain why polarizing sunglasses are especially good at blocking annoying reflections on wet roads an cars in front of you etc when driving.

   (c) Some transparent tape (*doorschijnende plakband*) is taped on top of the glasses of linearly polarizing sunglasses. This is repeated a few times, with pieces of tape of variable length taped to the glasses at random places and in random orientations. The tape material has the property (shared with many transparant organic materials) that it rotates the polarization direction of light when light passes through it. The amount of rotation (per distance traveled through the tape) depends moreover on the frequency of the light passing through it. Now I put on another pair of polarizing sunglasses (without tape) and look at the light of the sun simultaneously through both the taped sunglasses and the ones I'm wearing. I'm slowly rotating the sunglasses with the tape. Predict what I will see. (If this description of the setup is not clear, the demo will be available during the oral part of the exam.)

   (d) Explain: The earth's magnetic field protects us from evil from outer space.

   (e) Explain why energy loss due to radiation becomes increasingly a problem for particle accelerators operating at higher energies, and why it is worse for lighter particles such as electrons than for heavier particles such as protons.

   (f) Suddenly interrupting a current can cause a spark, especially in circuits with a high self-inductance. Explain why. And while we're at it, what is a spark, physically?

   (g) Explain rainbows (qualitatively).

   (h) Say that, in our reference frame, a relatively nearby star A exploded today at noon and another relatively nearby star B, several light years away from star A, will explode tonight at 8pm. Does there in principle exist another inertial frame (observer) for whom the sequence of events is opposite, i.e. star B explodes before star A?

2. Imagine you have magnetic dipoles arranged on a chess board which lies in the xy plane. If the dipoles are constrained to point in the $\pm z$ directions, which of the following describes a minimum energy state of the system.

   (a) The dipoles all align along the $+z$ direction

   (b) The dipoles on the white squares point up, while de dipoles on the black squares point down.

   (c) The energy is the same regardless of how the dipoles are oriented.

   (d) It is too complicated to tell without numerical simulation.

3. Consider the following four potentials, function of time coordinate t and spacial coordinates $\mathbf{r} = (x, y, z)$:

   (a) $\phi = 0$; $\mathbf{A} = 0$ for $|x| < ct$ and $\mathbf{A} = \frac{\mu_0 k}{4c}(ct - |x|)^2 \hat{\mathbf{z}}$ for $|x| < ct$.

   (b) $\phi = 0$; $\mathbf{A} = -\frac{1}{4\pi\epsilon_0}\frac{qt}{r^2}\hat{\mathbf{r}}$.

   (c) $\phi = 0$; $\mathbf{A} = A_0 \sin(kx - \omega t)\,\hat{\mathbf{y}}$.

   (d) $\phi = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$; $\mathbf{A} = 0$.

   Are any of the four potentials physically equivalent to each other? (Justify you answer.)

4. A plasma, such as that found in the earth's ionosphere, consists of a gas of electrons of mass m_e and charge $-e$, which are free to move against a background of much heavier positive ions whose motion can be neglected. There are N_e electrons per unit volume. Assume that the density is uniform and that the interactions between the electrons may be neglected. Electromagnetic plane waves (angular frequency ω, wave number k) are incident on the plasma.

   (a) Find the conductivity σ as a function of ω.

   (b) Find the dispersion relation, i.e. find the relation between k and ω.

   (c) Find the index of refraction as a function of ω. Express n in terms of the plasma frequency, defined by $\omega_p^2 = N_e e^2/m_e\epsilon_0$.

   (d) What happens if $\omega < \omega_p$? During the day, the sun's ionizing radiation produces a density of free electrons $N_e \approx 3 \times 10^{12}\text{m}^{-3}$. Can you estimate an upper bound on radio frequencies used for long distance (e.g. transcontinental) communication? What do you think happens at night?

   (e) (*This is a hard problem and is optional; you can't lose points but it can give you extra credit.*) Now suppose there is an external magnetic field $\mathbf{B_0} = B_0\hat{\mathbf{z}}$. Consider a circularly polarized plane wave traveling parallel to $\mathbf{B_0}$. Show that the index of refraction is different for right and left circularly polarized waves. Assume that the magnetic field of the traveling wave itself is negligible compared to $\mathbf{B_0}$ (Recall: a circularly polarized wave traveling in the $\mathbf{z}$ direction means that the $\mathbf{x}$ component of the field leads or lags the $\mathbf{y}$ component by a phase of $\pi/2$.)