Examen elektrodynamica: 25 juni 2015

1 Oral question

Consider a spaceship flying with a speed v past two stations A and B. The stations are a distance d apart. When the spaceship flies past station A, it sends a light signal to station B. When the signal arrives, station B start measuring the time until the spaceship arrive at station B. Call the measured time τ.

1. What is τ?
2. What is the time t, according to an observer on the spaceship, it took for the spaceship to go from A to B.
3. Why is τ not appropriate to explain time dilation? Give a better time interval to explain time dilation.
   (*this question is mostly for discussion*)

2 Written questions

2.1 Question 1

We consider a ball $\mathcal{B}$. In its complement $\mathbb{R}^3 \setminus \mathcal{B}$ we have $\mathbf{B} = \mathbf{0}$. Let us have a subset $\mathcal{V} \subset \mathcal{B}$ that contains no sources ($\rho = 0$, $\mathbf{J} = \mathbf{0}$) and no time-dependent electrical fields ($\frac{\partial \mathbf{E}}{\partial t} = \mathbf{0}$). If $\mathcal{V} = \mathcal{B}$, argue that $\mathbf{B} = \mathbf{0}$ in $\mathcal{V}$.

Discuss whether and how this conclusion can change if $\mathcal{V}$ is taken to be

1. $\mathcal{B}$ with a smaller ball removed from it.
2. $\mathcal{B}$ with two smaller balls removed from it.
3. $\mathcal{B}$ with a tube removed from it. The tube connects the north and south pole of $\mathcal{B}$.
4. $\mathcal{B}$ with a donut removed from it.

2.2 Question 2

A double plate condensator is kept at a fixed potential difference by a battery. We insert a dielectric material with $\epsilon > \epsilon_0$ in between. As we do that, the system exerts a force on the dielectric material.

1. Is this force trying to pull or expel it? Explain via a qualitative comparison of energies of configurations differing by the area of the dielectric material that is inserted.

2. Allow the system to lower its energy by pushing or pulling the material (without acceleration). Where is the energy dissipated and what happens to the total charge on the plates?

2.3 Question 3

Consider an infinite conducting plane that is earthed (kept at a fixed potential) and bring a dipole $\mathbf{p}$ consisting of two opposite charges q, $-q$ a distance d apart, at a distance a from the plane, such that the dipole is parallel with the plane.

1. What is $\mathbf{E}$ on the side free of charges.

2. Argue that on the other side of the plane (i.e. the side with the charges) the leading contribution to $\mathbf{E}$ at a distance $r >> d$ and $r >> a$ from the plane is a quadrupole. (*if you cannot than assume this is true and go on*)

3. The charges induce a charge distribution ρ in the conducting plate. Assume that somehow one manages to copy that ρ onto an insulating substrate and take away the conductor. What is the monopole and dipole of ρ.

4. What would change in your answers to the above questions if the original dipole $\mathbf{p}$ had not been parallel to the plane.

2.4 Question 4

A perfectly insulating ball of radius r and mass m is given a total charge Q uniformly distributed over the ball. This charge distribution remains fixed on the ball whatever happens. We take two such balls, originally an infinite distance apart and we very slowly bring them closely together. This requires an amount of work W. We then release the balls. By symmetry they move in the exact opposite direction. Eventually the balls will cease feeling each other and both will move asympottically with speed v.

1. Argue that almost no radiation was emitted during the process of bringing the balls together.

2. Note m the rest mass before the charge Q was put on the ball. Calculate the rest mass m_* after Q was applied (that is of the object + its E.M. field).

3. Find the total radiated energy W_r as the balls are moving apart in terms of v and W. Do not neglect relativistic effects.

4. What happens if Q and r stay fixed while m drastically increases. Explain briefly.